Choosing Among Rules of k Names

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Abstract

Rules of k names are frequently used methods to appoint individuals to office. They are two-stage procedures where a first set of agents, the proposers, select k individuals from an initial list of candidates, and then another agent, the chooser, appoints one among those k in the list. The list of k names is often arrived at by letting each of the proposers vote for a fixed number q of candidates, and then choosing the k most voted ones. We can then speak of q-rukes of k names. We study the performance of q-rules of k names from two complementary perspectives. One of them focuses on the strategic behavior of agents operating under these rules, for each specific state of the world. In that direction, we provide partial characterization results for the strong Nash equilibria of the games induced by v-rules of k names. Our second perspective builds on what we learn about equilibria for each stage, but addresses a more aggregate question: what is the performance of each of these rules "in expected terms"? More specifically, we characterize the utilitarian and egalitarian optimal parameters of these rules (k,v) as functions of players´ degrees of risk aversion. The paper generalizes and extends results in our first exploration of the topic (Barberà and Coelho, 2010).

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1 Introduction

Appointing people to office is one of the main ways how the powerful exert their influence in society. But the ability to appoint officers is often limited in democracies.

Even the President of the United States has to submit his proposals for cabinet members, for supreme court judges and for many other appointments to the approval of the legislators. In other types of societies, or for other types of appointments, the power to choose one's candidate for a post may be almost unlimited. But even this power is often challenged.

In many historical circumstances, different groups have fought and competed for the ability to appoint people to important positions. The history of the Roman Church is full of instances where the secular rulers and the clergy have struggled to decide who had the possibility of appointing new bishops. In many European countries, University Rectors have been appointed sometimes by the Government, sometimes by the University community itself, sometimes by combinations of inputs from both.

In this paper we study a class of methods that allows several agents to share the power to appoint. These methods are widely used in the present , and were also used in the past. We call them rules of k names, and they work as follows. The set of deciders is divided into two groups: the proposers and the choosers. Proposers consider the set of all candidates to a position and screen k of them. Then, the choosers pick the appointee out of these k names.

The use of rules of k names is pervasive. Screening a few candidates before one is finally chosen is the standard practice in recruiting for all kinds of jobs. Many institutions choose their officers from a short list. Sometimes the use of these methods is mandated by law, while in other cases it is just the result of an agreement. And there are many different reasons for these two-stage procedures to be used. They allow for a division of labor that may be based on different degrees of expertise. They also allow dividing the power to appoint between the different sides. In this paper we emphasize the latter: we shall think of proposers and of choosers as two different groups, whose relative power may vary depending on the exact form of the rule.

Indeed, rules of k names can vary, depending on the composition of the sets of proposers and choosers, on the value of k, and also on the rules that the different participants adopt when deciding how to choose a list of candidates, or one candidate among many.

In this paper we make a number of simplifying assumptions that are, in fact, also adopted in many practical cases. The first one is that we consider only the case where the chooser is a single agent. This excludes the case where the choice from a restricted list is still a collective matter. But it is in fact a characteristic of many actual rules, as it is often the case that the chooser is one single person, like the president of a country, or some government appointed representative. Our second simplifying assumption is that proposers, when screening a list of k names, use a voting rule in the following form: each one submits a list of q candidates (for $v < k$), and then the k most voted candidates get into the list. Again, one can think of more general methods to select the k names, but the ones we consider are simple and frequently used.

Our rules are therefore fully specified once we have the number of potential candidates, the number of proposers, the size k of the list and the number v of votes that each proposer can cast.

Our purposes are twofold. On the one hand, we would like to understand the intricacies of the decision making process that will take place, under any given rule and for every specific society, as defined by the preferences of different proposers and those of the chooser. After a brief consideration of the simple case where there is only one proposer, we study the cooperative game arising when there are several proposers, and analyze the characteristics of its strong Nash equilibria. For the most general case, we cannot achieve a complete characterization of equilibria, nor reach an unequivocal existence result. Yet, our analysis, in the form of partial results and a variety of examples, is quite rich. It shows how sophisticated can the behavior of agents be under these rules, it identifies those variables that are crucial to identify the potential equilibria, and to understand the types of balance of power arising between the proposers and the chooser. In addition, we study some specific cases of interest for which we can guarantee the existence of equilibria and to characterize them. Even if limited, these special cases will prove useful for our second purpose.

Our second purpose is to evaluate the performance of different rules of k names from an ex ante point of view, by computing the utility that different participants in the decision process may expect. This would allow us to eventually arbitrate among different proposals for specific rules of k names, on the basis of their "average" performance and of the "average" satisfaction they can provide to different parts of society. For that purpose, we need to make different modelling decisions. One is on how to measure the utility of individuals, and how to calculate the eventual expected utility. In the absence of additional information, we consider that agents have utility functions whose argument is the ranking of alternatives, and treat them as Von Neumann-Morgenstern utility functions over lotteries. Ordinarily, they all express that getting the first ranked is better than the second ranked alternative, second is better than third, etc...Since the choice of any rule under uncertain preferences associates this rule with a lottery over the rankings of the chosen candidates. Under different distributional assumptions on the possible preferences of agents, we can compute the expected ranking of the chosen candidate, as well as the expected utilities for the chooser and for the proposers. Then the cardinal aspects introduced by our choice of utility functions will allow us to examine the role of different attitudes toward risk on the evaluation of alternative choices of the parameters k and q that characterize them.

In addition to the choice of a distribution and of a Bernoulli utility function, these calculations entail another consideration, that enriches them but also makes them more complicated. These are considerations about the timing at which expected utility calculations are carried out. Our leading assumption there is that the preferences that individuals will have in any realization of society are unknown by the planner, but that agents will have complete information about their preferences and those of their opponents at choice time, and will thus vote strategically. Because of that, in our main scenario, the outcome to be taken into account is the equilibrium outcome corresponding to each realization of the preferences of proposers and choosers.

A second scenario that we consider, for comparison with the full information one, is that where agents will be ignorant of the preferences of the rest of players, and just know their own preferences at play time.

Since the equilibrium analysis of the general case is quite ambiguous, we concentrate attention in two cases that, although limited in scope, provide quite a richness of results. One is the case of homogeneous proposers, i.e., the one where all proposers share the same preferences. With some qualifications, this case is very similar to the one where the proposer would be a single individual. The second special case is that of polarized societies, i.e., those where a group of proposers hold the same opinion, and the rest of them exhibit the opposite preferences. For these two cases, strong Nash equilibria for our games always exist

Our results on the expected utility consequences of choosing a given q-rule of k names over others allows us to compare the different choice of rules from an ex ante perspective, and to investigate two different questions.

One question relates to the choice of one of the roles, as a chooser or as a proposer, that individuals would make if allowed to. Here, there is a first mover advantage for the proposer, who can also count on a smaller variance in the distribution of chosen candidates. Large enough values of k can counteract this advantage. As a result, we can identify those utility classes and parameter values under which all agents would prefer to be the proposers, all would prefer to be the choosers, as well as cases where different agents would prefer to take different roles. The other use of our expected utility calculations is to perform some normative analysis. In the homogeneous case, choosing a rule that maximizes the weighted sum of expected utilities of the chooser with that of one proposer is equivalent to choosing one that equalizes as much as possible the weighted expected utilities of proposers and choosers. For the polarized case, the comparison is more complex.

The paper is organized as follows. In the next section, we formally define the v-rules of k names and we present two key parameters of screening rules that will allow us to partially characterize the equilibria of our games, as they are related to the sizes of coalitions with decision powewr. In Section 3 we discuss the game induced by v-rules of k names and we analyze the characteristics of its strong Nash equilibria. Then, in Section 4 we discuss two salient special cases where equilibria always exist, and we shall use in the following Section. Section 5, then, contains our expected utility calculations for different v-rules of k names under different alternative hypothesis, the discussion regarding the individual preferences over the role of chooser and proposer, and that of normative choice of rules. Brief concluding remarks follow in Section 6.

2 Rules of k names. The role of screening rules.

In this section we formally define rules of k names. We observe that, in addition to other structural features, like the number of proposers, the number of candidates and the size k of proposed candidates, a full specification of a rule of k names also requires to define the screening rules by which the proposers decide what names go into the list. In principle, this method could remain unspecified, or be rather complicated. But in actual practice simple and well specified screening rules are set. Basically, proposers are allowed to vote for a number v of candidates, and then the v most voted ones are selected (with a tie break if needed). Because of this, the section is devoted to formalize this simple family of screening rules, and to study how we can (partially) measure the power of individuals and coalitions when it come to choose a list of k names.

Notation 1 *Let* $\mathbf{A}$ *be the finite set of candidates. We denote by* $\mathbf{A}_k \equiv \{B \subseteq \mathbf{A} | \#B = k\}$ *the set of all possible subsets of* $\mathbf{A}$ *with cardinality* k *where* $\#B$ *stands for the cardinality of* B *and* $B \subseteq \mathbf{A}$ *means that* B *is contained in* $\mathbf{A}$*. Denote by* $\mathbf{N} \equiv \{1, ..., n\}$ *the finite set of committee members, the proposers, that selects a set* B *from* $\mathbf{A}_k$ *from which an individual that does not belong to* $\mathbf{N}$*, the chooser, selects a candidate for the office.*

Notation 2 *Let* W *be the set of all strict orders (transitive*[1]*, asymmetric*[2]*, irreflexive*[3] *and complete*[4]*) on* $\mathbf{A}$*. Each member* $i \in \mathbf{N} \cup \{chooser\}$ *has a strict preference* $\succ_i \in W$*. For any nonempty subset* B *of* $\mathbf{A}, B \subseteq \mathbf{A} \backslash \emptyset$*, we denote by* $\alpha(B, \succ_i) \equiv \{x \in B | x \succ_i y$ *for all* $y \in B \backslash \{x\}\}$ *the preferred candidate in* B *according to preference* $\succ_i$ *.*

Definition 1 *Let* $M^{\mathbf{N}} \equiv M_1 \times ... \times M_n$ *with* $M_i = M_j = M$ *for all* $i, j \in \mathbf{N}$ *where* M *is the space of actions of a proposer in* $\mathbf{N}$*. Given* $k \in \{1, 2, ..., \#\mathbf{A}\}$*, a* ***screening rule for k names*** *is a function* $S_k : M^{\mathbf{N}} \longrightarrow \mathbf{A}_k$ *associating to each action profile* $m_{\mathbf{N}} \equiv \{m_i\}_{i \in \mathbf{N}} \in M^{\mathbf{N}}$ *the k-element set* $S_k(m_{\mathbf{N}})$*.*

In words, a screening rule for k names is a voting procedure that selects k alternatives from a given set, based on the actions of the proposers. These actions may consist of single votes, sequential votes, the submission of preference of rankings, the filling of ballots, etc...For example, if the actions in $M^{\mathbf{N}}$ are casting single votes then $M \equiv \mathbf{A}$. If the actions in $M^{\mathbf{N}}$ are submissions of strict preference relation then $M \equiv W$.

Definition 2 *The* ***rule of k names*** *can be described as follows: given a set of candidates for office, a committee chooses k members from this set by using a screening rule for k*

[1] Transitive: For all $x, y, z \in A : (x \succ y$ and $y \succ z)$ implies that $x \succ z$.

[2] Asymmetric: For all $x, y \in A : x \succ y$ implies that $\neg(y \succ x)$.

[3] Irreflexive: For all $x \in A, \neg(x \succ x)$.

[4] Complete: For all $x, y \in A : x \neq y$ implies that ($y \succ x$ or $x \succ y$).

names. Then a single individual from outside the committee selects one of the listed names for the office.

Once we have these general definitions, we can become more specific. A we have already observed, it is usual in practice to specify the number of votes that each proposer can cast, and then use plurality. Our next definitions refers to this particular and important subclass of screening rules, and to the rules of k names that use them. We shall concentrate on this class of rules from now on.

Definition 3 *A screening rule for k names is a $v-$**votes screening rule for k names** if it can be described as follows: each proposer votes for v candidates and the list has the names of the k most voted candidates, with a tie breaking rule when needed. The parameters v and k satisfy $v \leq k < \#A$ and $v.n \geq k$ and the tie breaking criterion is established by strict ordering of alternatives.*[5]

Definition 4 *The $v-$**rule of k names** can be described as follows: given a set of candidates for office, a committee chooses k members from this set by using a $v-$votes screening rule for k names. Then a single individual from outside the committee selects one of the listed names for the office.*

In a preceding paper (Barberà and Coelho, 2010) we already noticed that there is a substantial difference between rules of k names, depending on the power that screening rules assign to majorities. Specifically, there are screening rules where the majority can always impose the full list, if it agrees to do so, and others where its power is more limited. We now introduce a formal definition that marks this difference, and remark on its implications in the case of v-rules of k names.

Definition 5 *(Barberà and Coelho, 2010) We say that a screening rule $S_k : M^{\mathbf{N}} \longrightarrow \mathbf{A}_k$ is **majoritarian** if and only if for every set $B \in \mathbf{A}_k$ there exists $m \in M$ such that for every strict majority coalition $C \subseteq N$ and every profile of the complementary coalition $m_{\mathbf{N}\setminus C} \in M^{\mathbf{N}\setminus C}$ we have that $S_k(m_{\mathbf{N}\setminus C}, m_C) = B$ provided that $m_i = m$ for every $i \in C$.*

[5] Notice if the number of candidates with positive votes is lower than k, the tie breaking criterion need to be used to break the ties among candidates with zero vote.

In words, we say that a screening rule is majoritarian if and only if for every set with k candidates there exists an action such that every strict majority coalition of proposers can impose the choice of this set provided that all of its members choose this action.

Remark 1 *Notice that among all v-rules of k names, those with v=k use majoritarian screening rules, and for any other v, they are non-majoritarian. Also notice that the definition of majoritarian screening rule states that it is enought that all proposers in the majoritarian coalition adopt the same strategy to exert their power. Clearly, this property can be met by the majoritarian k-rules for k names, since it will suffice that all proposers vote for the same set. As we shall see, under non-majoritarian screening rules, proposers will have to exert their power by coordinating on more complex combinations of votes.*

The concept of a majoritarian screening rule is an example of how we may approach a definition of the power of different sets of proposers. This will be an important consideration for our general analysis. Indeed, when a rule is majoritarian, any majority coalition can guarantee that, if it wants, it can impose the full list. In more general terms, we can inquire, for any rule, whether majoritarian or not, what is the size of coalitions that can indeed guarantee the inclusion of a full list, if its members agree upon it. Similarly, we can also look for milder forms of power. For example, by determining what coalitions can at least guarantee the inclusion of one candidate of their choice within the list.

We shall now introduce two parameters that one can define for any screening rule for k names, and that will be useful to characterize equilibria in the strategic game that we study in the coming section. Notice that, in general terms, these parameters will depend very much on the screening rules to be used. Luckily, in the case of v-rules of k names, they become easy to compute, and we provide an explicit formula for their values. Notice that our definitions are closely linked to that of effectivity functions studied by, among others, Peleg (1984), Abdou and Keiding (1991) and Sertel and Sanver (2004). These concepts of effectivity refer to the ability of agents to ensure an outcome, under the given rule.

Definition 6 *Given a screening rule for k names* $S_k : M^{\mathbf{N}} \longrightarrow \mathbf{A}_k$, *let* q_1 *be the minimum* $\widehat{q}$ *such that for any* $x \in A$ *and any coalition* $C \subseteq N$ *of voters with* $\#C \geq \widehat{q}$ *implies that there exists* $m_C \in M^C$ *such that for every profile of the complementary coalition* $m_{\mathbf{N}\setminus C} \in M^{\mathbf{N}\setminus C}$ *we have* $x \in S_k(m_C, m_{\mathbf{N}\setminus C})$.

In words, q_1 is the minimum $\widehat{q}$ such that for every candidate there exists an action profile such that any coalition with size higher or equal to $\widehat{q}$ can impose the inclusion of this candidate among the k chosen candidates.

Definition 7 *Given a screening rule for k names $S_k : M^{\mathbf{N}} \longrightarrow \mathbf{A}_k$, let q_k be the minimum $\widehat{q}$ such that for any $B \in A_k$ and any coalition $C \subseteq N$ of voters with $\#C \geq \widehat{q}$ implies that there exists $m_C \in M^C$ such that for every profile of the complementary coalition $m_{\mathbf{N}\backslash C} \in M^{\mathbf{N}\backslash C}$ we have that $S_k(m_C, m_{\mathbf{N}\backslash C}) = B$.*

In words, q_k is the minimum $\widehat{q}$ such that for every set with k candidates any coalition, with size higher or equal to $\widehat{q}$, can impose the choice of this set.

Remark 2 *$\frac{q_k}{k} \leq q_1 \leq q_k \leq n$, $q_1 > n - q_k$ and $q_k \geq \lceil \frac{n+1}{2} \rceil$, where $\lceil \frac{n+1}{2} \rceil$ stands for the superior integer of $\frac{n+1}{2}$. In particular, if the screening rule is majoritarian then $q_1 = q_k = \lceil \frac{n+1}{2} \rceil$.*

Our next proposition provides the explicit formulas for our parameters, in the case of v-votes screening rules.

Proposition 1 *If a screening rule for k names is a $v-$votes screening rule for k names then $q_k = \lceil \frac{kn}{(k+v)} \rceil + \mathcal{I}(\lfloor \frac{v \lceil \frac{kn}{(k+v)} \rceil}{k} \rfloor \leq n - \lceil \frac{kn}{(k+v)} \rceil)$ and $q_1 = \lceil \frac{vn}{(k+v)} \rceil + \mathcal{I}(\frac{vn}{(k+v)} = \lceil \frac{vn}{(k+v)} \rceil)$, where $\mathcal{I}$ denotes the indicator function.*

The proofs of the propositions are in the Appendix.

Remark 3 *The parameter q_k is a non-increasing function on the parameter v of the $v-$votes screening rule for k names and non-decreasing function on k.*

Remark 4 *The parameter q_1 is a non-decreasing function on the parameter v in the $v-$votes screening for k names and non-increasing function on k.*

Example 1 *Here we apply Proposition 1 to compute the parameters q_1 and q_k for all six screening rule for k names documented by Barberà and Coelho (2010). We begin by the four that belong to the family of v-votes screening rule for k names.*

1) Screening 3 names by 3-votes screening rule for 3 names: Each proposer votes for three candidates and the list has the names of the three most voted candidates, with a tie-break *when needed. It is used in the election of Irish Bishops and that of Prosecutor-General in most of Brazilian states.*

$q_1 = \lceil \frac{n+1}{2} \rceil \quad q_k = \lceil \frac{n+1}{2} \rceil$

2) Screening 5 names by 3-votes screening rule for 5 names: Each proposer votes for three candidates and the list has the names of the five most voted candidates, with a tie breaking rule when needed. It is used in the election of the members of Superior Court of Justice in Chile.

$q_1 = \lceil \frac{3n}{8} \rceil + \mathcal{I}(\frac{3n}{8} = \lceil \frac{3n}{8} \rceil) \quad q_k = \lceil \frac{5n}{8} \rceil + \mathcal{I}(\lfloor \frac{3\lceil \frac{5n}{8} \rceil}{5} \rfloor \leq n - \lceil \frac{5n}{8} \rceil)$

3) Screening 3 names by 2-votes screening rule for 3 names: Each proposer votes for two candidates and the list has the names of the three most voted candidates, with a tie breaking rule when needed. It is used in the election of the members of Court of Justice in Chile.

$q_1 = \lceil \frac{2n}{5} \rceil + \mathcal{I}(\frac{2n}{5} = \lceil \frac{2n}{5} \rceil) \quad q_k = \lceil \frac{3n}{5} \rceil + \mathcal{I}(\lfloor \frac{2\lceil \frac{3n}{5} \rceil}{3} \rfloor \leq n - \lceil \frac{3n}{5} \rceil)$

4) Screening 3 names by 1-vote screening rule for 3 names: Compute the plurality score of the candidates and include in the list the names of the three most voted candidates, with a tie breaking rule when needed. It is used in the election of rectors of public universities in Brazil.

$q_1 = \lceil \frac{n}{4} \rceil + \mathcal{I}(\lceil \frac{n}{4} \rceil = \lceil \frac{n}{4} \rceil) \quad q_k = \lceil \frac{3n}{4} \rceil + \mathcal{I}(\lfloor \frac{\lceil \frac{3n}{4} \rceil}{3} \rfloor \leq n - \lceil \frac{3n}{4} \rceil)$

The other two screening rules for k names documented by Barberà and Coelho (2010) do not belong to the family of v-votes screening rules for k names, but they are majoritarian:
5) 1-vote sequential plurality: The list is made with the names of the winning candidates in three successive rounds of plurality voting. It is used in the election of English Bishops.

$q_1 = \lceil \frac{n+1}{2} \rceil \quad q_k = \lceil \frac{n+1}{2} \rceil$

6) Screening 3 names by 3-vote sequential strict plurality: this is a sequential rule adopted by the Brazilian Superior Court of Justice to select its members. Each proposer votes for three candidates from a set with six candidates, and if there are three candidates with more votes than half of the total number of voters, they will form the list. It is used in the election of the members of the Brazilian Superior Court of Justice.

$q_1 = \lceil \frac{n+1}{2} \rceil \quad q_k = \lceil \frac{n+1}{2} \rceil$

The bounds established in definitions 6 and 7 are not as tight as they could be, because they are common to all alternative sets or candidates. We have proposed them for clarity, but we can now tightehn them a bit more, in order to take into account that tie-breaking rules or other assymetries that can change the bounds depending on the alternative or the set under consideration. This is the purpose of the next two defintions and remarks.

Definition 8 *Given a screening rule for k names* $S_k : M^{\mathbf{N}} \longrightarrow \mathbf{A}_k$ *and* $x \in A$*, let* $q_1(x)$ *be the minimum* $\widehat{q}$ *such that for any coalition* $C \subseteq N$ *of voters with* $\#C \geq \widehat{q}$ *implies that there exists* $m_C \in M^C$ *such that for every profile of the complementary coalition* $m_{\mathbf{N}\setminus C} \in M^{\mathbf{N}\setminus C}$ *we have* $x \in S_k(m_C, m_{\mathbf{N}\setminus C})$*.*

In words, $q_1(x)$ is the minimum $\widehat{q}$ such that there exists an action profile such that any coalition with size higher or equal to $\widehat{q}$ can impose the inclusion of x among the k chosen candidates.

Definition 9 *Given a screening rule for k names* $S_k : M^{\mathbf{N}} \longrightarrow \mathbf{A}_k$ *and* $X \in A_k$*, let* $q_k(X)$ *be the minimum* $\widehat{q}$ *such that for any coalition* $C \subseteq N$ *of voters with* $\#C \geq \widehat{q}$ *implies that there exists* $m_C \in M^C$ *such that for every profile of the complementary coalition* $m_{\mathbf{N}\setminus C} \in M^{\mathbf{N}\setminus C}$ *we have that* $S_k(m_C, m_{\mathbf{N}\setminus C}) = X$*.*

In words, $q_k(X)$ is the minimum $\widehat{q}$ such that for any coalition, with size higher or equal to $\widehat{q}$, can impose the choice of X.

Remark 5 *Consider any v-votes screening rule for k names and any* $x \in A$*, if* x *is one the k-top candidates according to the tie breaking criterion then* $q_1(x) = q_1$ *or* $q_1(x) = q_1 - 1$ *. If* x *is not one of k-top candidates according to the tie breaking criterion then* $q_1(x) = q_1$*.*

Remark 6 *Consider any v-votes screening rule for k names and any* $X \in A_k$*, if the set* X *is formed by the k-top candidates according to the tie breaking criterion then* $q_k(X) = q_k$ *or* $q_k(X) = q_k - 1$*. If* X *is not formed by the k-top candidates according to the tie breaking criterion then* $q_k(X) = q_k$*.*

3 The constrained chooser game

In this section, we model a game that represents the possible strategic interaction among the proposers, in view of their preferences and those of the chooser, and we analyze its strong Nash equilibria. Although we cannot provide a full characterization of such equilibria in the most general case, the picture that emerges from our result in one of richness, regarding the strategic possibilities that are open to the different proposers. We start by providing a set of necessary conditions that any equilibrium must fulfill (Proposition 2). Then we provide an example of the way in which these conditions help us to locate an equilibria (Example 2). Yet, we must acknowledge that these conditions are not sufficient, as shown by Example 5. Moreover, the role of tie breaking rules is quite disturbing, since they may be crucial to determine whether the equilibrium is unique (Example 3), or even whether it exists (Example 6). Another bothersome feature is that the parity of the number of proposers does matter when discussing the uniqueness of equilibria (Example 4).

The above qualifications, and other comments arising along their discussion, are meant to emphasize that a full characterization of equilibria is not an easy matter. Yet other propositions in the section allow us to identify important cases where one can guarantee the existence and the uniqueness of equilibria. For example, if enough proposers agree with the chooser (Proposition 3), or if enough agreement among the proposers, as reflected in the existence of a Condorcet winner with qualified majority (propositions 4 and 5).

In addition to all these results and examples regarding equilibria, we can also offer some comparative static results on the role of parameter v (Proposition 6 and Example 7). Let us now define our game formally.

Definition 10 *(Barberà and Coelho, 2010) Given* $k \in \{1, 2, ..., \#\mathbf{A}\}$*, a screening rule for* k *names* $S_k : M^{\mathbf{N}} \longrightarrow \mathbf{A}_k$ *and a preference profile* $\succ \equiv \{\succ_i\}_{i \in \mathbf{N} \cup \{chooser\}} \in W^{\mathbf{N}+1}$*, the* ***Constrained Chooser Game*** *can be described as follows: It is a simultaneous game with complete information where each player* $i \in \mathbf{N}$ *chooses a strategy* $m_i \in M_i$*. Given* $m_{\mathbf{N}} \equiv \{m_i\}_{i \in \mathbf{N}} \in M^{\mathbf{N}}$*,* $S_k(m_{\mathbf{N}})$ *is the chosen list with* k *names and the winning candidate is* $\alpha(S_k(m_{\mathbf{N}}), \succ_{chooser})$*.*

In the Constrained Chooser Game, the chooser 's strategy set is restricted to a single element. In that sense, we could say that he is not an active player. Specifically, we take it that the chooser will simply select that candidate that is best for him among those that he will be presented with. Thus, the chooser's preferences will condition the outcome function, and therefore will have an impact on the equilibrium play of the proposers. But we exclude the possibility that he announces a choice rule that is not in accordance to his preferences, which are known in each game.

We choose to analyze the set of strong Nash equlibria of this game. This is consistent with the idea that proposers have complete information about their preferences and those of the chooser, and that they must find ways to cooperate among themselves, in order to come up with a favorable list.

Definition 11 *(Barberà and Coelho, 2010) Given $k \in \{1, 2, ..., \#\mathbf{A}\}$, a screening rule for k names $S_k : M^{\mathbf{N}} \longrightarrow \mathbf{A}_k$ and a preference profile $\succ \equiv \{\succ_i\}_{i \in \mathbf{N} \cup \{chooser\}} \in W^{\mathbf{N}+1}$, a joint strategy $m_{\mathbf{N}} \equiv \{m_i\}_{i \in \mathbf{N}} \in M^{\mathbf{N}}$ is a* ***pure strong Nash equilibrium of the Constrained Chooser Game*** *if and only if, given any coalition $C \subseteq N$, there is no $m'_{\mathbf{N}} \equiv \{m'_i\}_{i \in \mathbf{N}} \in M^{\mathbf{N}}$ with $m'_j = m_j$ for every $j \in \mathbf{N} \backslash C$ such that $\alpha(m'_{\mathbf{N}}, \succ_{chooser}) \succ_i \alpha(m_{\mathbf{N}}, \succ_{chooser})$ for each $i \in C$.*

Definition 12 *A candidate is a* ***chooser's ℓ-top candidate*** *if and only if he is among the ℓ best ranked candidates according to the chooser's preference.*

Definition 13 *A candidate $x \in \mathbf{B} \subseteq \mathbf{A}$ is a p-**Condorcet winner** over $\mathbf{B}$ if $\#\{i \in N | x \succ_i y\} \geq p$ for any $y \in \mathbf{B} \backslash \{x\}$[6]. In words, a candidate is the p-Condorcet Winner over a subset of $\mathbf{A}$ if and only if it is beats any other alternative that belongs to this subset by at least a p-majority. It is important to note that the chooser's preferences over candidates are not taken into account in this definition. We say that a candidate is a strong Condorcet winner if and only if $\frac{n}{2}-$Condorcet winner.*

Proposition 2 below provides necessary conditions of a candidate to be a strong Nash equilibrium outcome of the Constrained Chooser Game

Proposition 2 *Consider a v-rules for k names, if candidate x is a strong Nash equilibrium outcome of the Constrained Chooser Game then it satisfies the following three conditions*

1. *It is a chooser's ($\#\mathbf{A} - k + 1$)-top candidate.*

[6]Where $\#\{i \in \mathbf{N} | x \succ_i y\}$ stands for the cardinality of $\{i \in \mathbf{N} | x \succ_i y\}$ and $\mathbf{B} \subseteq \mathbf{A}$ means that B is contained in $\mathbf{A}$.

2. *If y is a chooser's ($\#A-k+1$)-top candidate then $\#\{i \in N | y \succ_i x\} < q_k(Y)$ for any $Y \in A_k$ such that y is the chooser best candidate in Y.*

3. *If y is the chooser's 1-top candidate then $\#\{i \in N | y \succ_i x\} < q_1(y)$.*

The following example illustrates how Proposition 2 can be useful for identifying the set of equilibrium outcomes.

Example 2 *Let $\mathbf{A} = \{a, b, c, d, e\}$ and let $\mathbf{N} = \{1, 2, 3\}$. Suppose that each proposer votes for one candidate and the three most voted candidates form the list , with a tie breaking rule when needed: $b \succ a \succ e \succ d \succ c$. .The preferences of the chooser and the committee members are as follows:*

Preference Profile

Proposer 1	*Proposer 2*	*Proposer 3*	*Chooser*
e	e	e	a
d	d	d	b
c	c	b	c
a	a	a	d
b	b	c	e

Notice that , we have that $q_1(x) = 1$ for any $x \in A$ and $q_k(X) = 3$ for any $X \in A_k$.
The first step in describing the equilibrium outcomes is to identify those candidates that satisfy the three necessary conditions established in Proposition 2.
Inspecting the preference profile above and recalling that $\#\mathbf{A} = 5$, we have that:
1. *Condition 1: $\{a, b, c\}$.*
2. *Condition 2: $\{a, b, c, d, e\}$.*
3. *Condition 3:$\{a, e, d\}$*

So, only candidate a that satisfies all three conditions. Now we have to check if there is a strategy profile that satisfies candidate a as a strong Nash equilibrium candidate. The following strategy profile sustains a as a strong Nash equilibrium outcome: Proposer 1 votes for a, Proposer 2 votes for d and Proposer 3 votes for b.
The table below presents the set of strong Nash equilibrium for different values of v. Notice that , in this example, the chooser is weakly worst off as v increases.

Set of strong Nash equilibrium outcomes

k=3 $v = 1$ $q_1 = 1$ $q_k = 3$ $\{a\}$

k=3 $v = 2$ $q_1 = 2$ $q_k = 3$ $\{a\}$

k=3 $v = 3$ $q_1 = 2$ $q_k = 2$ $\{c\}$

The example below show that the Constrained Chooser Game can have more than one strong Nash equilibrium outcome and they may depend on how ties are broken.

Example 3 *Let $A = \{a, b, c, d, e\}$, and let $N = \{1, ..., 11\}$. Each proposer votes for one candidate and the list has the names of the three most voted candidates, with a tie breaking rule when needed: $b \succ a \succ e \succ d \succ c$.*

Preference Profile

1 proposers type 1	*7 proposers type 2*	*3 proposers type 3*	*Chooser*
b	a	c	b
a	c	a	c
e	d	e	a
d	b	d	e
c	e	b	d

First, by Proposition 1, $q_1(x) = 3$ for any $x \in A$ and $q_k(X) = 9$ for any $X \in A_k$. First, notice that candidates a and c satisfy all three necessary conditions stated in Proposition 2. Second, notice that the chooser prefers c to a and $\#\{i \in N | c \succ_i a\} \geq q_1(c)$. However, candidate a is still a strong Nash equilibrium outcome. Consider the following strategy profile that sustains a as a strong Nash equilibrium outcome: the seven type 2 proposers cast three votes for a, two votes for e, one vote for b and one vote for d. Type 1 proposer casts a vote for b, while the three type 3 proposers cast two votes for d and one for e. So, a,d and e will have three votes each, while b only two. Thus, the selected list is $\{a, d, e\}$ and a is the winning candidate. The readers can check that there is no coalition of voters that has incentive in deviating.

Now, consider the following strategy profile that sustains c as a strong Nash equilibrium outcome:the seven type 2 proposers cast four votes for a and three votes for e. Type 1 proposer casts a vote for b, while the three type 3 proposers cast three votes for c. Thus, the selected list is $\{a, c, e\}$ and c is the winning candidate. Again, the readers can check

that there is no coalition of voters that has incentive in deviating.
Here there is a intuition for these two equilibria: the equilibrium strategy that sustaining candidate a represents a clever way in which the type 2's distribute their votes and leave the type 3's not being able to select c, even if they all vote for it. Voters of type 2 ensure that candidate a, their favorite, is among the proposed ones, by casting three votes in its favor. They also give enough support to candidate b in such a way that, along with the vote of type 1, b is still not chose but would be as soon as there candidates with two votes that should enter the list. In view of the fact that b has two votes, proposers of type 3 cannot vote for their favorite, c, because if they all spent their votes on c, which would make c eligible, then some alternative with two votes would come in, and in this case it would be b, which they hate and is the chooser's best. Given that they cannot get c, they then concentrate , in alliance with type 2 people, in getting e and d into the list, above their worse alternative b, and at least get their secod alternative.
The other equilibrium, the one sustaining c, is more obvious: the type 3's go ahead, support c, and then the type 2's have to prevent b from coming by "wasting" their remaining votes by supporting e.
Thus, it is like the presence of one type 1 proposer voting for b leads both types 2 and 3 in a sort of race: if one of them uses the most rewarding strategy in one of the two equilibria, the other must chicken and concede. If both used their most rewarding strategies,then b, that they both hate, would come out! Notice also that if the tie breaking criterion was $a \succ e \succ d \succ b \succ c$*, then c would be the unique Strong Nash equilibrium outcome.*

Barberà and Coelho (2010) proved that if the screening rule is majoritarian and the number of proposers is odd then the set of strong Nash equilibrium outcome of Constrained Chooser Game is singleton or empty. In the example below the number of proposers is even and the Constrained Chooser Game has more than one equilibrium outcome.

Example 4 *Let* $\mathbf{A} = \{a, b, c, d\}$ *and let* $\mathbf{N} = \{1, 2\}$*. Suppose that each proposer votes for two candidates and the two most voted candidates form the list , with a tie breaking rule when needed:* $c \succ a \succ d \succ b$*. The preferences of the chooser and the committee members are as follows:*

	Preference Profile	
Proposer 1	*Proposer 2*	*Chooser*
b	c	a
c	b	b
d	d	c
a	a	d

Notice that $q_1(c) = q_1(a) = 1$, $q_1(b) = q_1(d) = 2$, $q_1(\{a,c\}) = 1$ *and* $q_k(X) = 2$ *for any* $X \in A_k \backslash \{a,c\}$.

Inspecting the preference profile above and applying Proposition 2, we have that:

1. *Condition 1:* $\{a,b,c\}$.
2. *Condition 2:* $\{b,c\}$.
3. *Condition 3:*$\{b,c\}$

So, only candidates b *and* c *satisfy all three conditions. Let us show that the set of strong Nash equilibrium outcomes is* $\{b,c\}$. *Consider the following strategy profile that sustains* c *as a strong Nash equilibrium outcome: Proposers casts votes for* c *and* d. *So, selected list is* $\{c,d\}$ *and the winning candidate is* c. *Now, consider the following strategy profile that sustains* b *as a strong Nash equilibrium outcome: Proposer 1 casts votes for* b *and* a *and Proposers 2 casts votes for* b *and* c. *So, selected list is* $\{b,c\}$ *and* b *is the winning candidate.*

The example below shows that the set of necessary conditions established by Proposition 2 are not sufficient conditions and the equilibrium may not exist.

Example 5 *Let* $\mathbf{A} = \{a,b,c,d\}$ *and let* $\mathbf{N} = \{1,2,3\}$. *Suppose that each proposer votes for one candidate and the two most voted candidates form the list, with the following tie breaking rule when needed:* $a \succ c \succ b \succ d$. *The preferences of the chooser and the committee members are as follows:*

	Preference Profile		
Proposer 1	*Proposer 2*	*Proposer 3*	*Chooser*
c	b	b	a
b	c	a	c
d	d	c	b
a	a	d	d

We have that $q_1(a) = q_1(c) = 1$, $q_1(x) = 2$ for any $x \in A\backslash\{a, c\}$ and $q_k(X) = 2$ for any $X \in A_k$ such that $a \in X$ and $q_k(Y) = 3$ for any $Y \in A_k$ such that $a \notin Y$. Inspecting the preference profile above, we have that:

1. *Condition 1: $\{a, b, c\}$.*
2. *Condition 2: $\{b, c\}$.*
3. *Condition 3:$\{b\}$*

So, only candidate b satisfies the three necessary conditions stated in Proposition 2. However, b is not an equilibrium outcome. He is not an equilibrium outcome, since Proposer 1 always have incentive in preventing the election of b by casting vote in c.
Notice also that the proposers preference profile satisfies single peakedness, so this example teaches us that this property does not guarantee existence of an equilibrium. If we had considered a 2-votes screening rules for two names, candidate b would be the unique strong Nash equilibrium outcome of the game.The table below presents the set of strong Nash equilibrium for different values of v.

Set of strong Nash equilibrium outcomes

k=2	$v = 1$	ϕ
k=2	$v = 2$	$\{b\}$

The example below shows that the existence of strong Nash equilibrium outcome may depend on the tie breaking criterion.

Example 6 *Let $A = \{a, b, c, d\}$, and let $N = \{1, ..., 3\}$. Each proposer votes for one candidate and the list has the names of the two most voted candidates, with a tie breaking rule when needed: $c \succ a \succ b \succ d$.*

Preference Profile

Proposer 1	*Proposer 2*	*Proposer 3*	*Chooser*
a	c	b	a
b	a	c	b
c	b	a	c
d	d	d	d

We have that $q_1(a) = q_1(c) = 1$, $q_1(x) = 2$ for any $x \in A\backslash\{a, c\}$ and $q_k(X) = 2$ for any $X \in A_k$ such that $c \in X$ and $q_k(Y) = 3$ for any $Y \in A_k$ such that $c \notin Y$. Inspecting the preference profile above, we have that:

1. *Condition 1:* $\{a, b, c\}$.
2. *Condition 2:* $\{a, c\}$.
3. *Condition 3:*$\{a\}$

Thus, only candidate a satisfies all three necessary conditions stated in Proposition 2. Consider the following strategy profile that sustains a as a strong Nash equilibrium outcome: each proposer casts a vote for his second highest ranked candidate, that is Proposer 1 votes for b, Proposer 2 votes for a and Proposer 3 votes for c. Thus, the selected list is $\{c, a\}$ and a is the winning candidate. Only proposers 2 and 3 would have incentive in changing the equilibrium outcome. Neither of them alone can change the outcome in favor of their favorite candidates. By making a joint deviation, they would just be able to induce the victory of b, but Proposer 2 would be worst off in this case. Notice that if the tie breaking criterion was $c \succ d \succ b \succ a$, the set of strong Nash equilibrium outcome would be empty. The same would happen if the screening rule was 2 votes screening rule for two names.

Set of strong Nash equilibrium outcomes

k=2	$v = 1$	$\{a\}$
k=2	$v = 2$	ϕ

Proposition 3 *Consider any $v-$rule for k names and let x be one of the chooser´s $(\#A-k+1)$-top candidates and $X \in A_k$ such that x is the chooser best candidate in X, if $q_k(X)$ proposers rank x highest then x is the unique strong Nash equilibrium outcome of the Constrained Chooser Game.*

Proposition 4 *Consider any $v-$rule for k names, if a candidate x is $n-\lfloor \frac{nv}{2k} \rfloor+1-$Condorcet winner over the set of chooser's $(\#\mathbf{A} - k + 1)$-top candidates then it is the unique strong Nash equilibrium outcome of the Constrained Chooser Game.*

Proposition 5 *Consider any $v-$ rule for k names and denote by x chooser´s 1-top candidate. if x is also a $q_1(x)-$Condorcet winner over the set of chooser's $(\#\mathbf{A} - k + 1)$-top candidates then it is the unique strong Nash equilibrium outcome of the Constrained Chooser Game.*

To close the section, we present some comparative statics results, that allow us to understand some of the consequences of choosing one value of v over another.

Proposition 6 *If the chooser 1-top-candidate is a strong Nash equilibrium outcome of the Constrained Chooser Game under $\acute{v}-$rule for k names then it is also a strong Nash equilibrium outcome of the Constrained Chooser Game under any $\widetilde{v}-$ rule for k names whenever $\widetilde{v} < \acute{v}$ provided that both screening rules have the same tie breaking criteria.*

Surprisingly, Example 7 shows that the chooser can be worse off under 1-vote screening rule for 2 names than under 2-vote screening rule for 2 names.

Example 7 *Let $A = \{a, b, c, d\}$, and let $N = \{1, 2, 3\}$. Each proposer votes for one candidate and the list has the names of the two most voted candidates, with a tie breaking rule when needed: $c \succ d \succ b \succ a$.*

Preference Profile

Proposer 1	*Proposer 2*	*Proposer 3*	*Chooser*
b	a	a	b
d	c	c	a
c	d	d	c
a	b	b	d

As can be verified, c is the unique equilibrium outcome of the Constrained Chooser Game under the 1-vote screening rule for 2 names.

Here there is a intuition for this result: notice that candidate a cannot be a strong equilibrium outcome of the Constrained Chooser Game, because as long as proposer 1 votes for b, proposers 2 and 3 cannot get a to be the outcome, even if they can force a to be in the list. Short of that, proposers 2 and 3 coordinate their actions so that one of them votes for c and the other for d. If 1 persists in voting for b, this creates a tie between the three candidates that is solved in favor of c and d, out of which the chooser selects c. If 1 votes for c instead, the same outcome ensues. And all other actions by any combination for agents would lead some of them to outcomes that would be worse than c for some of them. Hence, c is the unique strong Nash equilibrium of the Constrained Chooser Game under our proposed rule.

Now let us change the screening rule for 2 names. Suppose that the proposers use 2-votes screening rule for 2 names. Now, a is the strong Nash equilibrium outcome of the Constrained Chooser Game. This shows that here the chooser is better off under 2-vote screening rule for 2 names than under the 1-vote screening rule for 2 names.

4 Two special cases with simple solutions

In this section we describe two special cases of societies based on restrictions over the structure of the proposer's preference profiles. They share the property of guaranteeing the existence of well characterized equilibria. This makes them especially useful when providing expected utility calculations in the following section.

4.1 The homogeneous proposers model

First, consider the case where all proposers share the same preferences. The analysis of this simple case is similar, though not identical, to that of the special case when there is one single proposer. This is why we start by considering that even more special case.

If there is only one proposer, using a backward induction rationality, the equilibrium outcome is the best alternative for the proposer out of the $a - k + 1$-top alternatives of the chooser.

Now take any v-voting rule of k names. It is clear that the whole set of proposers, acting together, can always guarantee a proposal that determines whatever set of k names the proposer decide to send to the chooser (notice, however, that this may involve different proposers voting for different candidates. After this observation, the analysis of this case proceeds as that of the one proposer case.

Proposition 7 *Consider the homogeneous proposers model. The strong Nash equilibrium outcome is the best alternative for the proposers out of the $a - k + 1$-top alternatives of the chooser.*

4.2 The polarized proposers model

Now consider the case where two groups of agents hold identical preferences within the group and opposite to those of proposers in the other side. An important result is that under the assumptions of what we call the Polarized Proposers Model, strong Nash equilibria always exist for the corresponding restricted choosers game, and the equilibrium outcome is unique. This result will help us in our search for expected utility evaluation of alternative rules, to be carried out in Section 5. Moreover, it represents a first example of special models for which we can guarantee existence of equilibria under non majoritarian screening rules. Hopefully, other interesting settings with these characteristics will arise.

Assumptions on the preferences profile:

1. (Assumption 1). There are two groups of proposers, denoted by G_1 and G_2, such that $G_2 = N \backslash G_1$.

2. (Assumption 2). All the proposers in G_1 share the same preferences over the set of candidates.

3. (Assumption 3). All the proposers in G_2 share the same preferences over the set of candidates and it is the reverse of the preferences of the proposers in G_1.

4. (Assumption 4). The tie breaking rule coincides with at least one of the agent´s preferences over the set of candidates.

Denote by m the cardinality of majoritarian group of proposers, so if $\#G_1 \geq \#G_2$ then $m = \#G_1$, otherwise $m = \#G_2$.

Proposition 8 *Consider the Polarized Proposers Model, odd number of proposers and any v-rule for k names. A strong Nash equilibrium outcome of the Constrained Chooser Game always exists and it is unique. In addition:*
1) Suppose that the tie breaking criterion coincides with the chooser´s preferences over the set of candidates or with the minoritarian group ´s preferences over the set of candidates.
If $m \geq q_k$ then the strong Nash equilibrium outcome is the best alternative of individuals in the majoritarian group out of chooser´ s $(\#\mathbf{A} - k + 1)$-top alternatives;
If $q_k > m$ then the strong Nash equilibrium outcome is the chooser´s 1-top alternative.

2) Suppose that the tie breaking criterion coincides with the majoritarian group´s preferences over the set of candidates.
If $m \geq q_k > n - m$ then the strong Nash equilibrium outcome is the best alternative of individuals in the majoritarian group out of chooser´ s $(\#\mathbf{A} - k + 1)$-top alternatives;
If $q_k > m \geq q_1 > n - m$ then the strong Nash equilibrium outcome is the chooser´s best alternative out of the majoritarian group´s k-top candidates;
If $q_k > m > n - m \geq q_1$ then the equilibrium outcome is the chooser´s top alternative.

The example below shows that without assumption 4 the Polarized proposers model may not has a strong Nash equilibrium outcome.

Example 8 *Let* $\mathbf{A} = \{a, b, c, d, e, f\}$ *and let* $\mathbf{N} = \{1, 2, 3\}$*. The proposers use 1-rule for 4 names with the following tie breaking rule when needed:* $e \succ d \succ c \succ b \succ a \succ f$*. The preferences of the chooser and the committee members are as follows:*

Preference Profile

Proposer 1	*Proposer 2*	*Proposer 3*	*Proposer 4*	*Proposer 5*	*Chooser*
d	d	d	d	f	f
e	e	e	e	b	a
c	c	c	c	a	b
a	a	a	a	c	c
b	b	b	b	e	d
f	f	f	f	d	e

First, by Proposition 1, $q_1 = 2$ *and* $q_k = 5$*. We have that* $q_1(x) = 1$ *for any* $x \in \{b, c, d, e\}$*,* $q_1(x) = 2$ *for any* $x \in A \backslash \{b, c, d, e\}$ *and* $q_k(X) = 5$ *for any* $X \in A_k \backslash \{b, c, d, e\}$ *and* $q_k(\{b, c, d, e\}) = 4$ *.Notice that proposers 1, 2,3, and 4 form the majoritarian group of proposers, so* $m = 4$*. Notice also that the tie breaking rule is equal to the reverser of chooser´s preference over the set of candidates. The first step in describing the equilibrium outcomes is to identify those candidates that satisfy the three necessary conditions established in Proposition 2.*
Inspecting the preference profile above and recalling that $\#\mathbf{A} = 6$*, we have that:*
1. Condition 1: $\{a, b, f\}$*.*
2. Condition 2: $\{a, b, c, d, e\}$*.*
3. Condition 3:$\{a, b, c, d, e, f\}$
So, only candidates a *and* b *satisfy all three conditions. However, there exists no strategy profile that can sustain them as an strong Nash equilibrium outcome of the Constrained Chooser Game.*

Corollary 1 *Consider the Polarized Proposers Model and odd number of proposers. The chooser cannot be worst off under* $\acute{v}$*-rule for k names than under* $\widetilde{v}-$ *rule for k names whenever* $\widetilde{v} > \acute{v}$*.*

The corollary above follows from Proposition 8 and by the fact that q_k is a decreasing function on v.

Corollary 2 *Consider the Polarized Proposers Model, odd number of proposers and $v-$rule of k names. The chooser cannot be worst off under a more polarized set of proposers (small m) than under a less polarized set of proposers (big m).*

The corollary above follows from Proposition 8.

Corollary 3 *Consider the Polarized Proposers Model and odd number of proposers. The chooser cannot be worst off under v- rule for $\acute{k}$ names than under $v-$ rule for $\acute{\acute{k}}$ names whenever $\acute{\acute{k}} > \acute{k}$.*

The corollary above follows from Proposition 8 and by the fact that q_k is a increasing function on k.

5 Comparison of payoff distributions in terms of returns and risks

In this section, we study the agents´s payoff distributions from applying different v-rules of k names. Based on these payoff distributions, we analyze the optimal k and the parameter v of the screening rule according to utilitarian and egalitarian criteria.

5.1 Homogeneous committee

In this subsection, we consider the Homogeneous Proposers Model and assume that agents´ preferences are random draw from an uniform distribution over the domain of preferences.

5.1.1 Distribution of the ranking of equilibrium outcome .

Let us assume that homogeneous proposers and the chooser´s preferences are the result of independent random draws from an uniform distribution over domain of strict preferences. Given that the agents´s preferences are random variables, the ranking of the equilibrium outcome according to the preferences of tone of the agents is also a random variable. Denote by R_c and R_p the random variables that represent the ranking of the equilibrium outcome according to the chooser and proposers´ preference relation and by r_c and r_p

the realized values which R_c and R_p may take, so $r_i = 1 + \#\{y \in N | y \succ_i x\}$ if x is the equilibrium outcome. Notice that if equilibrium outcome is the agent i´s best candidate then $r_i = 1$ and if it is the agent i´s worst candidate $r_i = a$.

The random variable R_p has the same distribution of the smallest element of a random sample with size $s = a - k + 1$ drawn without replacement from a population $D = \{1, 2, \ldots, a\}$ uniformly distributed. Thus, following the standard results of order statistics literature, we have:

Let describe R_p by means of the cumulative distribution function $F_p : \{1, .., a\} \rightarrow [0, 1]$. That is, for any r_p, let $F_p(r_p)$ be the probability that the realized ranking of the equilibrium outcome is less or equal than r_p according to the proposers´ preferences is :

$$F_p(r_p = x | a, k) = \begin{cases} \sum_{j=1}^{x} \frac{\binom{a-j}{a-k}}{\binom{a}{a-k+1}} & if \quad x \in \{1, ..., k\} \\ \\ 1 & \text{otherwise} \end{cases} \qquad (1)$$

Equations (2) and (3) below give the formulas of the mean and variance of R_p:

$$E(R_p | a, k) = \frac{a+1}{a-k+2} \qquad (2)$$

$$Var(R_p | a, k) = \frac{(a-k+1)(a+1)(k-1)}{(a-k+2)^2(a-k+3)} \qquad (3)$$

After some algebraic manipulation, we can see that $E(R_p | a, k)$ is strict increasing with k but at a decreasing rate with respect to k and $Var(R_c | k, a)$ is strict increasing with respect to k,but at a increasing rate with respect to k.

Let $F_c : \{1, .., a\} \rightarrow [0, 1]$ be the the cumulative distribution function of R_c.Notice that the random variable R_c has the same distribution of a discrete random variable uniformly distributed over $\{1, 2, \ldots, a - k + 1\}$. Therefore, we have that:

$$F_c(r_c = x | k, a) = \begin{cases} \frac{x}{a-k+1} & if \quad x \in \{1, ..., a - k + 1\} \\ \\ 1 & \text{otherwise} \end{cases} \qquad (4)$$

Equations (5) and (6) below give the formulas of the mean and variance $E(R_c)$.

$$E(R_c | k, a) = \frac{a-k+2}{2} \qquad (5)$$

$Var(R_c|k,a) = \frac{(a-k)(a-k+2)}{12}$ (6)

Notice that $E(R_c|k,a)$ is strict decreasing with respect to k at a constant rate and $Var(R_c|k,a)$ is strict decreasing with respect to k, but at a decreasing rate with respect to k.

Remark 7 *Notice that $E(R_c|k,a)E(R_p|a,k) = \frac{a+1}{2}$ for every $k \in \{1,...,a\}$.*

Example 9 *Suppose $a = 10$, Figure 1 and Figure 2 display the cumulative distributions of R_c and R_p under $k = 8$ (Figure 1) and $k = 4$ (Figure 2). Notice that in Figure 1 the distribution of R_c first order stochastically dominates the distribution R_p, while in Figure 2 the reverse holds. Figures 3 and 4 displays the mean and the variance of R_p and R_c for different values of k.*

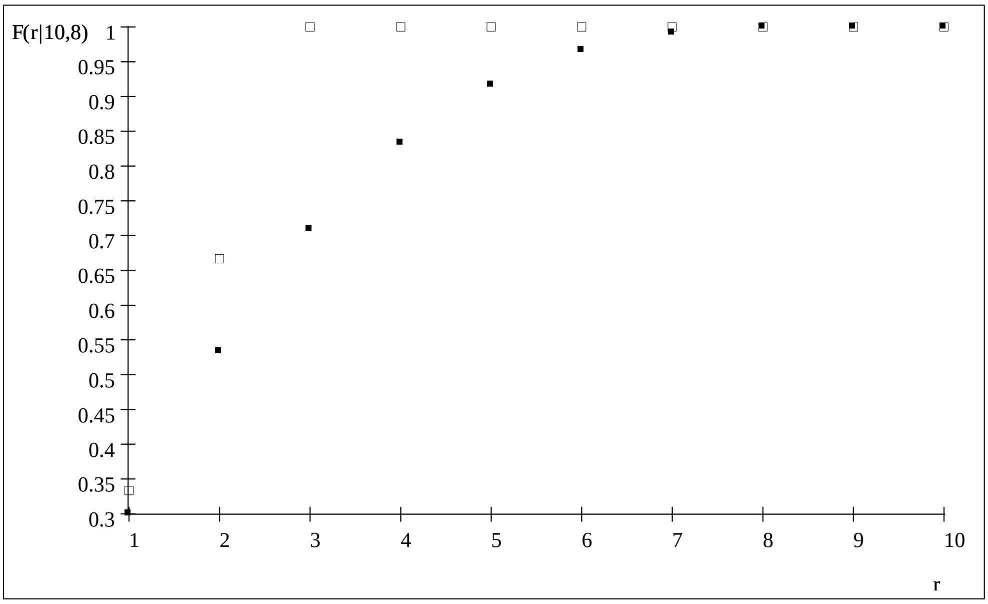


Figure 1: $F_p(r_p|10,8)$ black boxes and $F_c(r_c|10,8)$ white boxes.

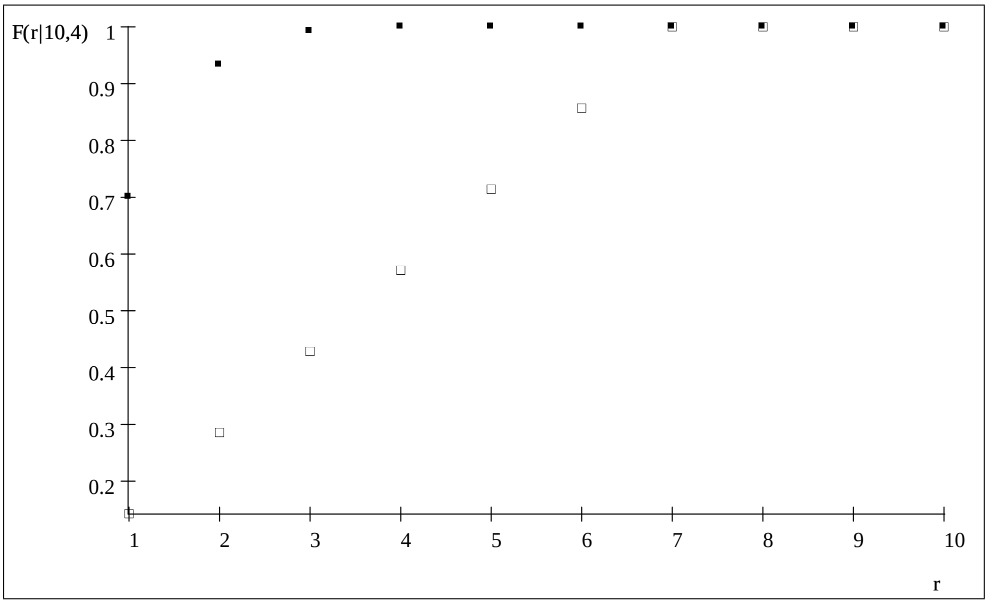


Figure 2: $F(r_p|10,4)$ black boxes and $F(r_c|10,4)$ white boxes.

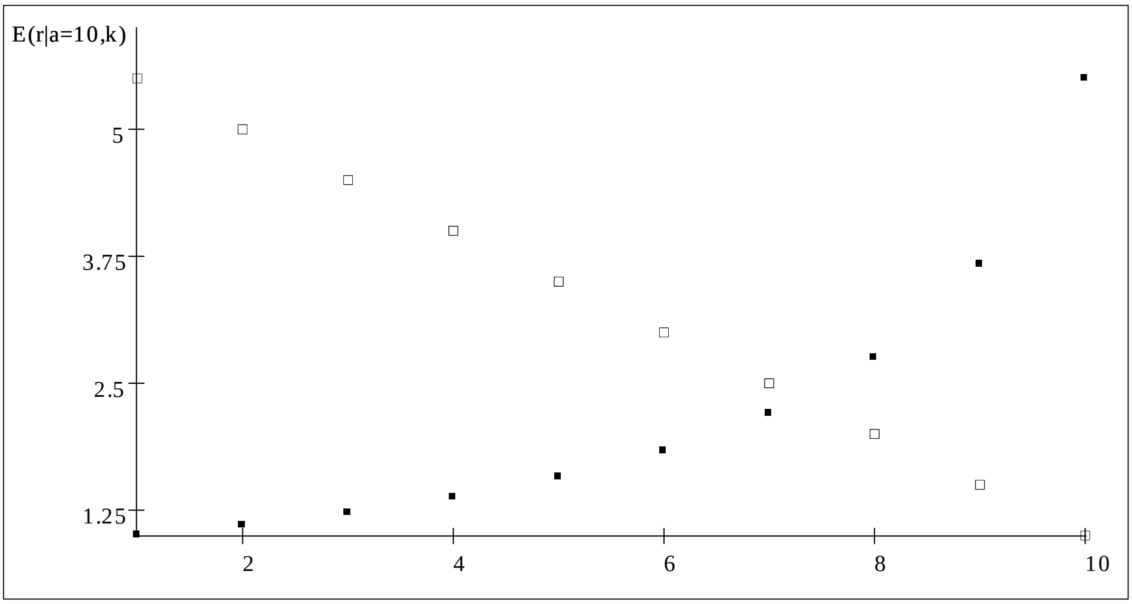


Figure 3: $E(R_p|a=10,k)$ black boxes and $E(R_c|a=10,k)$ white boxes.

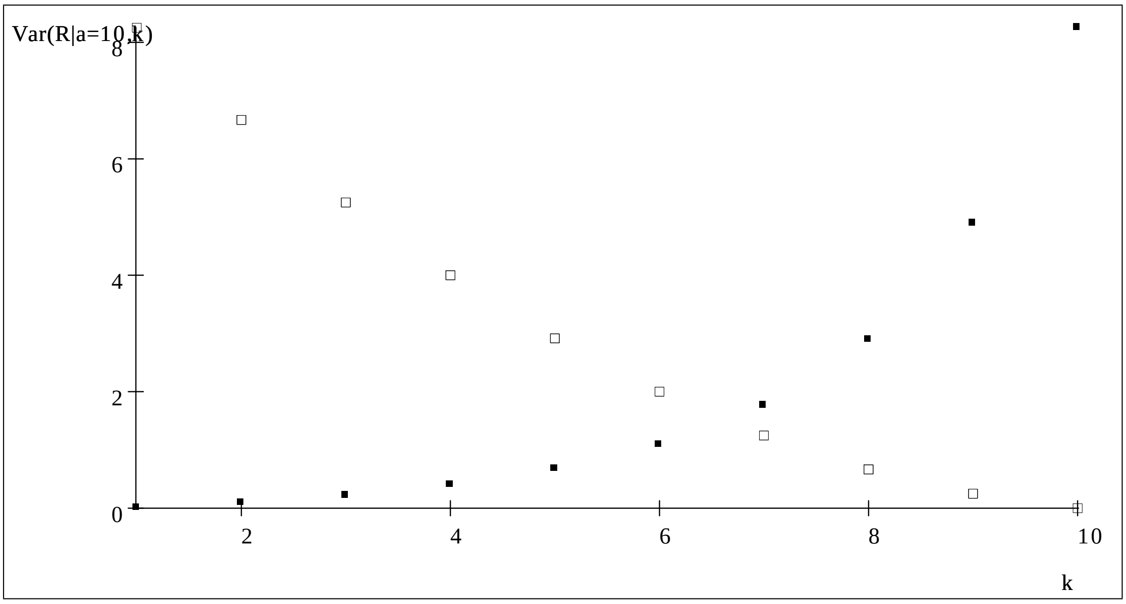


Figure 4: $Var(R_p|a = 10, k)$ black boxes and $Var(R_c|a = 10, k)$ white boxes.

Proposition 9 *Consider any number of candidates a:*

1) $E(R_c|k, a) > E(R_p|k, a)$ *for every* $k < a + 2 - \sqrt{2a + 2}$;

2) $E(R_c|k, a) = E(R_p|k, a)$ *if* $k = a + 2 - \sqrt{2a + 2}$ *is an integer number*;

3) $E(R_p|k, a) > E(R_c|k, a)$ *for every* $k > a + 2 - \sqrt{2a + 2}$.

The proposition follows from equations (4) and (6), the fact that if $k = a+2-\sqrt{2a + 2}$ then we have that $(\frac{a+1}{a-k+2}) = \frac{(a-k+2)}{2}$, $E(R_p|a, k)$ is strict increasing with k and $E(R_c|a, k)$ is strict decreasing with k.

5.1.2 Proposer or chooser: which one would you like to be?

The results presented in this subsection will try to help us to answer the following question: proposer or chooser: which position gives a higher expected payoff in the game induced by the rule of k names? We answer this question comparing the expected utility in each position of an imaginary agent i that has Bernoulli utility function, $u(r)$, that is strict decreasing with r. We will show that the answer will depend of the level of the curvature of $u(r)$, that is level of the risk aversion of the agent.

Example 10 *Consider $a = 17$ and $k = 12$. Suppose that agent i has the following decreasing and concave utility function $u(r) = -e^{\gamma r}$ where $\gamma > 0$. The parameter γ is the coefficient of absolute risk aversion. Proposer or Chooser: which position would give a higher payoff to agent i? Before answer this question, let us compute the expected returns and risks face by the proposer and chooser. Using the expressions (2),(3),(5) and (6), we have that:*

$E(R_p|a = 17, k = 12) = \frac{17+1}{17-12+2} = 2.5714; Var(R_p|a = 17, k = 12) = \frac{(17-12+1)(17+1)(12-1)}{(17-12+2)^2(17-12+3)} =$

$3.0306;$
$E(R_c|a = 17, k = 12) = \frac{17-12+2}{2} = 3.5; Var(R_c|a = 17, k = 12) = \frac{(17-12)(17-12+2)}{12} =$

$2.9167.$
Notice, under a=17 and k=12, if agent i was risk neutral, he would prefer to be the proposer since $E(R_p|a = 17, k = 12) < E(R_c|a = 17, k = 12)$.

However, agent i is risk averse, so he cares about the risks of being in each position. Let us assume that $\gamma = 0.2$. So, his expected utility in each position is: $E(u(R_p)|a = 17, k = 12) = \sum_{j=1}^{12} \frac{-e^{0.2j}\binom{17-j}{17-12}}{\binom{17}{17-12+1}} = -1.795\,3$ and $E(u(R_c)|a = 17, k = 12) = \sum_{j=1}^{17-12+1} \frac{-e^{0.2j}}{17-12+1} = -2.1332$.

Thus, since $E(u(R_p)|a = 17, k = 12) > E(u(R_c)|a = 17, k = 12)$, agent i prefers to be the proposer. However, if $\gamma = 0.8$ (that is more risk averse), he would prefer to be the chooser since $E(u(R_p)|a = 17, k = 12) = \sum_{j=1}^{12} \frac{-e^{0.8j}\binom{17-j}{17-12}}{\binom{17}{17-12+1}} = -41.514$ and $E(u(R_c)|a = 17, k = 12) = \sum_{j=1}^{17-12+1} \frac{-e^{0.8j}}{17-12+1} = -36.474$.

Proposition 10 below gives the values of k for which we do not need to have information of the functional form of the utility function, in order to know the best position (proposer or chooser) to play the game induced by the rule of k names (see Figure 9).

Proposition 10 *For any utility function $u(\cdot)$, we have that:*

1) $E(u(R_p)) > E(u(R_c))$ for every $k < \frac{a+1}{2}$;

2) $E(u(R_c)) > E(u(R_p))$ for every $k > a - \sqrt[2]{a} + 1$.

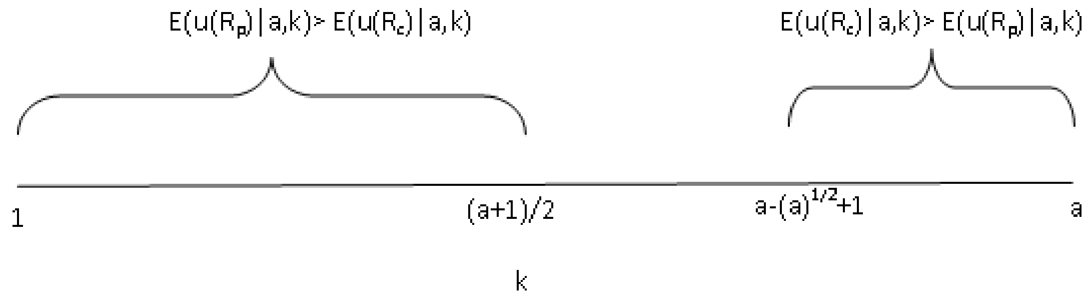


Figure 5: Illustration of Proposition 10.

In Figure 5, we can see that if $k \in [\frac{a+1}{2}, a - \sqrt{a} + 1]$, without knowing the utility function of an agent, we cannot know the best position (proposer or chooser) for him to play the game induced by the rule of k names.

The next proposition tells us that if we know whether the utility function is concave or convex, we can enlarge the interval of values of k that we know for sure what is the best position to play the game (see figures 6 and 7).

Proposition 11 *Consider any number of candidates a:*
1) For any strict decreasing and concave utility function $u(\cdot)$:
$E(u(R_c)) > E(u(R_p))$ *for every* $k > a + 2 - \sqrt{2a + 2}$.
2) For any strict decreasing and convex utility function $u(\cdot)$:
$E(u(R_p)) > E(u(R_c))$ *for every* $k < a + 2 - \sqrt{2a + 2}$.

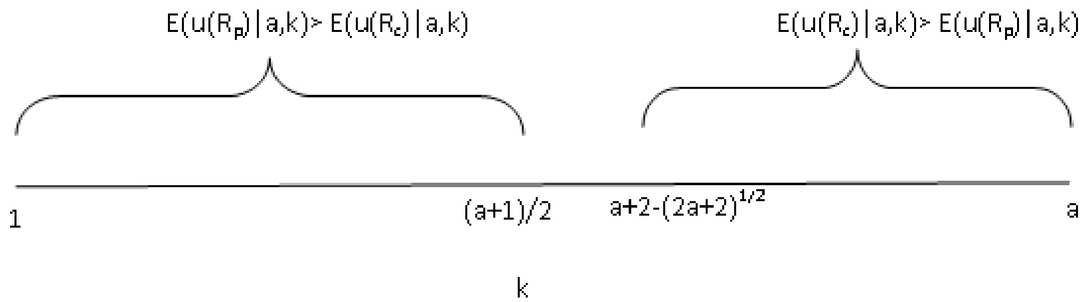


Figure 6: Illustration of Proposition 11 for the case where u( ) is concave.

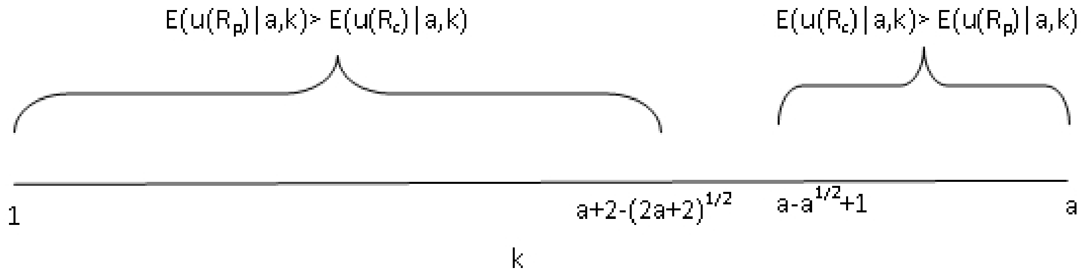


Figure 7: Illustration of Proposition 11 for the case where u( ) is convex.

The next proposition below show that if an agent has a linear utility function, for any possible value of k, we will know for sure what is the best position to play the game (see Figure 8).

Proposition 12 *For any linear utility function $u(\cdot)$:*

1) $E(u(R_p)|k,a) > E(u(R_c)|k,a)$ for every $k < a + 2 - \sqrt{2a+2}$;

2) $E(u(R_p)|k,a) = E(u(R_c)|k,a)$ if $k = a + 2 - \sqrt{2a+2}$ is an integer number;

3) $E(u(R_c)|k,a) > E(u(R_p)|k,a)$ for every $k > a + 2 - \sqrt{2a+2}$.

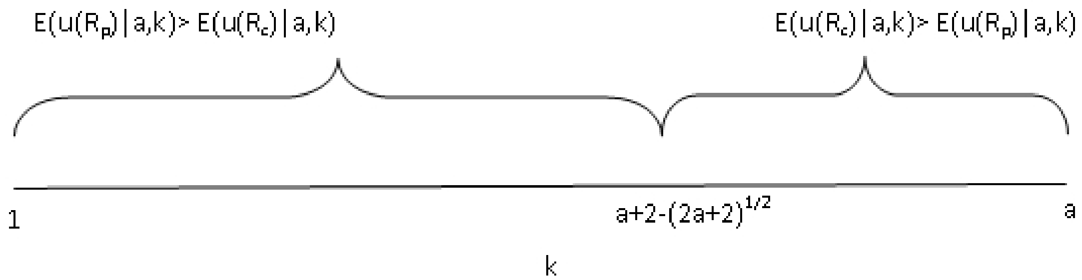


Figure 8: Illustration of Proposition 12, u( ) is linear.

Example 11 *Suppose that $a = 17$. Notice that $a + 2 - \sqrt{2a+2} = 13$. Thus, applying Proposition* 12*, we have that $E(R_p|a = 7, k = 13) = E(R_c|a = 7, k = 13)$. By expressions (4)-(7), we have that $E(R_p|a = 17, k = 13) = E(R_c|a = 17, k = 13) = 3$, $Var(R_p|a = 17, k = 13) = 4.2857$ and $Var(R_c|a = 17, k = 13) = 2$. Notice also that any risk averse agent would prefer to be the chooser than the proposer and any risk lover agent would prefer to be the proposer.*

5.1.3 Welfare analysis: The optimal k

In this subsection, we study the optimal k according to utilitarian and egalitarian criteria. Here we parameterize the chooser and proposers´ Bernoulli utility functions in order to study how the optimal k varies with the number of candidates and their level of risk aversion. Suppose the following standard functional form of Bernoulli utility function:

$$u_i(r_i) = -r_i - \gamma_i r_i^2 + \gamma_i \text{ where } \gamma_i > -\tfrac{1}{2a} \qquad (8)$$

Remark 8 *Notice that:*
1) Given that $\gamma_i > -\frac{1}{2a}$, $u_i(r_i)$ is strict decreasing with r_i.
2) If $\gamma_i = 0$ then $u_i(r_i) = -r_i$ (risk neutral).
3) If $\gamma_i > 0$ then $u_i(r_i)$ is strict concave with r_i (risk averse).
4) If $0 > \gamma_i > -\frac{1}{2a}$ then $u_i(r_i)$ is strict convex with r_i (risk lover)

Example 12 *Consider $a = 6$. The Figure 9 below plots $u_i(r_i)$ for three different values of $\gamma_i \in \{-1/13, 0, 1/13\}$.*

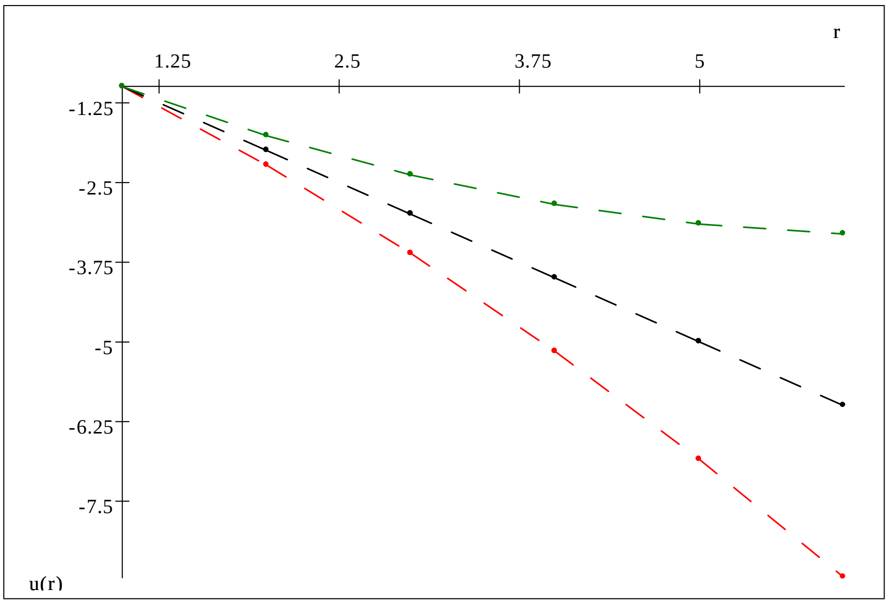


Figure 9: $\gamma = -1/13$ (red), $\gamma = 0$ (black) and $\gamma = 1/13$ (green)

Taking the expectation of both sides of expression (8), we have that:

$$E(u(r)|a,k) = -E(r|a,k) - \gamma(E(r^2|a,k)) + \gamma \qquad (9)$$

Given that $Var(r|a,k) = E(r^2|a,k) - E(r|a,k)^2$ and (9), we have that

$$E(u(r)|a,k) = -E(r|a,k) - \gamma(Var(r|a,k) + E(r|a,k)^2) + \gamma \qquad (10)$$

Therefore, after some algebraic manipulation with expressions (2)-(3), (5)-(6) and (10), we have the explicit formulas of the proposer and chooser´s expected utilities:

$$E(u_p(R_p)|a,k) = -\frac{(a+1)}{(a-k+2)} - \frac{\gamma_p(a+k+1)(a+1)}{(a-k+2)(a-k+3)} + \gamma_p \qquad (11)$$

$$E(u_c(R_c)|a,k) = -\frac{(a-k+2)}{2} - \frac{\gamma_c(a-k+2)(2a-2k+3)}{6} + \gamma_c \qquad (12)$$

Remark 9 *Notice that* $E(u_p(R_p)|a,k=1) = -1$, $E(u_c(R_c)|a,k=1) = -\frac{(a+1)}{2} - \frac{\gamma_c(2a+5)(a-1)}{6}$, $E(u_p(R_p)|a,k=a) = -\frac{(a+1)}{2} - \frac{\gamma_p(2a+5)(a-1)}{6}$ *and* $E(u_c(R_c)|a,k=a) = -1$. *Thus,* $E(u_p(R_p)|a,k=1) > E(u_c(R_c)|a,k=1)$ *and* $E(u_c(R_c)|a,k=a) > E(u_p(R_p)|a,k=a)$ *since* $\gamma_c > \frac{1}{-2a}$ *and* $\gamma_p > \frac{1}{-2a}$.

Example 13 *Consider* $a=7$, $\gamma_p = \gamma_c = 0$ *(risk neutral). Figure 10 plots the expected utilities for different values of k. Notice that for any* $k \leq 4$, *the proposers have a higher expected utility than the chooser and when k=5 they have the same expected utilities.*

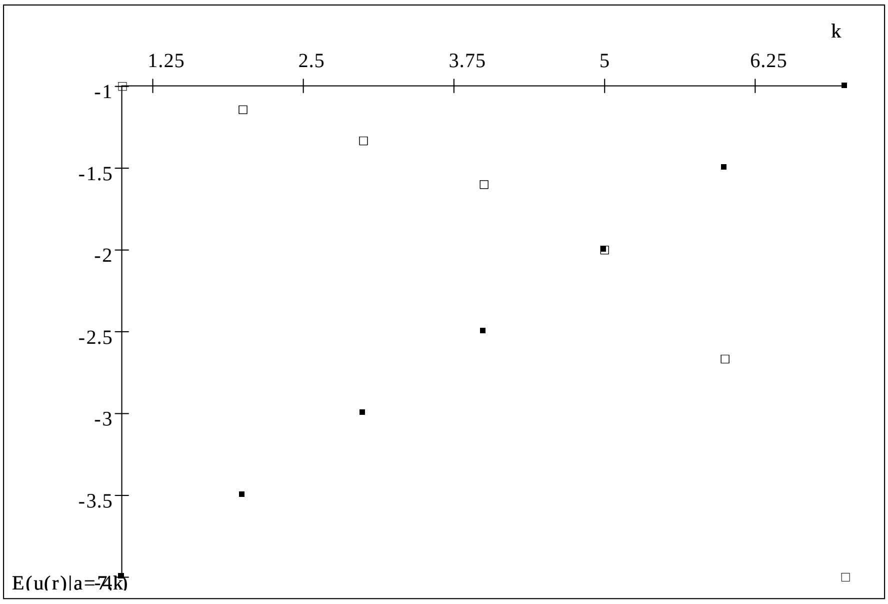


Figure 10: $E(u_p(r_p)|a,k)$ *(white box) and* $E(u_c(r_c)|a,k)$ *(black)*

Suppose now that the proposers are risk averses. Figure 11 plots the expected utilities when $\gamma_p = 3, \gamma_c = 0$ *and* $a = 7$. *Notice that the chooser has a higher expected utility than the proposers even when* $k = 3$.

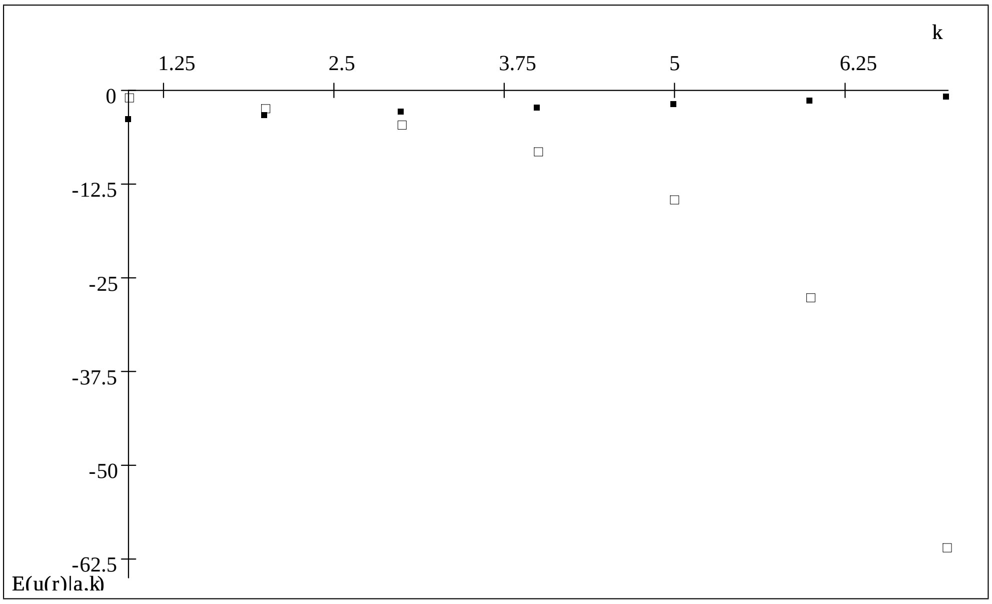


Figure 11: $E(u_p(r_p)|a,k)$ (white) and $E(u_c(r_c)|a,k)$ (black)

Proposition below states that there is threshold in the interval $t \in [1, a]$ such that for for any value of $k < t$ the proposers have higher payoffs than the chooser and for any $k > t$ than the reverse holds. Moreover the threshold is only a function of a, k, γ_p and γ_c.

Proposition 13 *For any values of $\gamma_p \geq \frac{1}{-2a}$ and $\gamma_c \geq \frac{1}{-2a}$ there is $a > t(a, \gamma_c, \gamma_p) > 1$ such that:*

1) $E(u_p(R_p)|k, a) > E(u_c(R_c)|k, a)$ for every $k < t(a, \gamma_c, \gamma_p)$;

2) $E(u_p(R_p)|k, a) = E(u_c(R_c)|k, a)$ if $k = t(a, \gamma_c, \gamma_p)$ is an integer number;

3) $E(u_c(R_c)|k, a) > E(u_p(R_p)|k, a)$ for every $k > t(a, \gamma_c, \gamma_p)$.

In particular, $t(a, \gamma_c, \gamma_p)$ has a simple formula when $\gamma_p = \gamma_c = 0$:

$t(a, \gamma_c = 0, \gamma_p = 0) = a + 2 - \sqrt{2a + 2}$

Proposition 13 follows from the fact $E(u_p(r)|a,k)$ is strict decreasing with k, $E(u_c(k)|a,k)$ is strict increasing with k and $E(u_p(r)|a, k = 1) > E(u_c(r)|a, k = 1)$ and $E(u_c(r)|a, k = a) > E(u_p(r)|a, k = a)$. We find the explicit form of the function $t(a, \gamma_c, \gamma_p = 0)$ by solving the following equation: $-\frac{(a+1)}{(a-k+2)} = -\frac{(a-k+2)}{2}$.

Proposition 14

1) If $\gamma_p \geq \gamma_c > 0$ *then* $1 < t(a, \gamma_c, \gamma_p) < a + 2 - \sqrt{2a+2}$.

2) If $\gamma_p \leq \gamma_c \leq 0$ *then* $a + 2 - \sqrt{2a+2} < t(a, \gamma_c, \gamma_p) < a$.

3)If $\gamma_c \geq \frac{3(\sqrt[2]{2a+2}-1)^2}{(2a+1)}\gamma_p > 0$ *then* $a + 2 - \sqrt{2a+2} < t(a, \gamma_c, \gamma_p) < a$.

4) If $\gamma_c \leq \frac{3(\sqrt[2]{2a+2}-1)^2}{(2a+1)}\gamma_p < 0$ *then* $1 < t(a, \gamma_c, \gamma_p) < a + 2 - \sqrt{2a+2}$.

Corollary 4 *If* $\gamma_p = \gamma_c = \gamma > 0$ *then* $t(a, \gamma_c = 0, \gamma_p = 0) > t(a, \gamma_c = \gamma, \gamma_p = \gamma)$.
If $\gamma_p = \gamma_c = \gamma < 0$ *then* $t(a, \gamma_c = \gamma, \gamma_p = \gamma) > t(a, \gamma_c = 0, \gamma_p = 0)$.

Example 14 *Suppose that* $a = 1000$. *Here we compute* $t(a, \gamma_c, \gamma_p)$ *for different values of* γ_c *and* γ_p. *We have that* $t(a, \gamma_c = 0, \gamma_p = 0) = 1000 + 2 - \sqrt{2000+2} = 957.26$, $t(a, \gamma_c = 500, \gamma_p = 500) \simeq 952.9$ *and* $t(a, \gamma_c = -1/3000, \gamma_p = -1/3000) \simeq 957.36$, $t(a, \gamma_c = 0, \gamma_p = 500) \simeq 190.5$ *and* $t(a, \gamma_c = 500, \gamma_p = 0) \simeq 999.429$. *It seems that* $\frac{t(a,\gamma_c,\gamma_p)}{t(a,\gamma_c=0,\gamma_p=0)}$ *is close to 1 when* $\gamma_p = \gamma_c = \gamma$.

Definition 14 *A* $k \in \{1, ..., a\}$ *satisfies the egalitarian solution if* $E(u_p(R_p)|k, a) - E(u_c(R_c)|k, a)|$ $\leq |E(u_p(R_p)|\acute{k}, a) - E(u_c(R_c)|\acute{k}, a)|$ *for every* $\acute{k} \in \{1, ..., a\}$. *We denote by* $S_e(a, \gamma_c, \gamma_p)$ *the set of ks that satisfy the egalitarian solution. We denote* $k_e(a, w, \gamma_c, \gamma_p)$ *the largest k in* $S_e(a, \gamma_c, \gamma_p)$.

Definition 15 *A* $k \in \{1, ..., a\}$ *satisfies the utilitarian solution if* $wE(u_p(R_p)|k, a) + (1 - w)E(u_c(R_c)|k, a) \geq wE(u_p(R_p)|\acute{k}, a) + (1 - w)\frac{E(u_c(R_c)|\acute{k},a)}{n}$ *for every* $\acute{k} \in \{1, ..., a\}$ *where* $w \in (0, 1)$. *We denote by* $S_u(a, \gamma_c, \gamma_p)$ *the set of ks that satisfy the utilitarian solution. We denote* $k_u(a, \gamma_c, \gamma_p)$ *the largest k in* $S_u(a, \gamma_c, \gamma_p)$.

Proposition 15 *There exist at most two values of k that maximizes* $wE(u_p(R_p)|k, a) + (1 - w)E(u_c(R_c)|k, a)$. *In the case where k and* $k\,'$ *both maximize* $wE(u_p(R_p)|k, a) + (1 - w)E(u_c(R_c)|k, a)$, *k and* $k\,'$ *are adjacent, i.e.* $k = k\,' - 1$. *The utilitarian* $k_u(a, w, \gamma_c, \gamma_p)$

is equal to the largest $\acute{k} \in \{1, ..., a\}$ *such that* $\frac{(a+1)(a-k'+4)+\gamma_p(3a+k'+4)(a+1)}{(a-k+3)(a-k'+2)(4a\gamma_c-4k'\gamma_c+9\gamma_c+3)} \geq \frac{(1-w)}{w}\frac{1}{6}$ *and* $\frac{(a+1)(a-k'+4)+\gamma_p(3a+k'+4)(a+1)}{(a-k+3)(a-k'+2)(4a\gamma_c-4k'\gamma_c+9\gamma_c+3)} \leq \frac{(1-w)}{w}\frac{1}{6}$ *for any* $k > \acute{k}$.

Moreover if $\frac{(a+1)(a-k'+4)+\gamma_p(3a+k'+4)(a+1)}{(a-k'+3)(a-k'+2)(4a\gamma_c-4k'\gamma_c+9\gamma_c+3)} = \frac{(1-w)}{w}\frac{1}{6}$ *then* $\acute{k}$ *is a twin-dips.*
In particular, k_u *has a simple formula when* $\gamma_p = \gamma_c = 0$ *:*

$$k_u(a, w, \gamma_c = 0, \gamma_p = 0) = \left\lfloor a + \frac{5}{2} - \sqrt{\frac{w}{1-w}\left(2a + \frac{7w+1}{4w}\right)} \right\rfloor$$

Proposition 16 *Let* $\gamma_p = \gamma_c = 0$. $k \in \{1, ...a\}$ *minimizes* $|E(u_p(R_p)|a,k) - E(u_c(R_c)|a,k))|$ *if and only if maximizes* $E(u_p(R_p)|a,k) + E(u_c(R_c)|a,k)$.

Corollary 5 below states that, when $\gamma_p = 0, \gamma_c = 0$ and $w = \frac{1}{2}$, the utilitarian k is not lower than $\frac{(a+1)}{2}$ and is equal to the egalitarian k.

Corollary 5 *Let* $\gamma_p = 0, \gamma_c = 0$ *and* $w = \frac{1}{2}$*:*
$k_u(a, w = \frac{1}{2}, \gamma_c = 0, \gamma_p = 0) = k_e(a, \gamma_c = 0, \gamma_p = 0) = \left\lfloor a + \frac{5}{2} - \sqrt{2a + \frac{9}{4}} \right\rfloor \geq \frac{(a+1)}{2}$.

Remark 10 *Let* $\gamma_p = 0, \gamma_c = 0$ *and* $k^* = a + \frac{5}{2} - \sqrt{2a + \frac{9}{4}}$. *Notice that* $k^* - t(a, \gamma_c = 0, \gamma_p = 0) = \sqrt{2a+2} + \frac{1}{2} - \sqrt{2a + 2 + \frac{1}{4}}$. *Thus* $1 > k^* - t(a, \gamma_c = 0, \gamma_p = 0) > 0$ *for every* $a > 0$. *Therefore if* $t(a, \gamma_c = 0, \gamma_p = 0)$ *is an integer number we have that* $\left\lfloor a + \frac{5}{2} - \sqrt{2a + \frac{9}{4}} \right\rfloor = t(a, \gamma_c = 0, \gamma_p = 0)$.

Example 15 *Suppose that* $a = 1000$, $\gamma_p = 0$, $\gamma_c = 0$ *and* $w = \frac{1}{2}$. *Applying Proposition 11 and Corollary 5,* $k_u(a, w = \frac{1}{2}, \gamma_c = 0, \gamma_p = 0) = k_e(a, \gamma_c = 0, \gamma_p = 0) = \lfloor a + (5/2) - \sqrt[2]{(2a + 9/4)} \rfloor = 957$. *Under* $k = 957$, *the expected ranking of the equilibrium outcome is* $(a+1)/(a-k+2) = 22.244$ *according to the proposers preferences and* $(a-k+2)/2 = 22.5$ *according to the chooser´s preferences.*

The example below shows that Corollary 5 does not apply for the case where $\gamma_p \neq \gamma_c$.

Example 16 *Suppose that* $a = 7$. *Here, we compute the optimal* k *when* $\gamma_c = 0$ *and* $\gamma_p = 0.5$: $k_u(a, w = \frac{1}{2}, \gamma_c = 0, \gamma_p = 0.5) = 2$ *and* $k_e(a, \gamma_c = 0, \gamma_p = 0.5) = 4$.

The example below shows that the equivalence between utilitarian and egalitarian criteria when $\gamma_p = \gamma_c = 0$ seems to hold even for the case where $\gamma_p = \gamma_c \neq 0$, but it remains to be proved.

Example 17 *Suppose that* $a = 1000$. *Here, we compute* $k_u(a, w = \frac{1}{2}, \gamma_c, \gamma_p)$ *and* $k_e(a, \gamma_c, \gamma_p)$ *for* $(\gamma_c = 500, \gamma_p = 500)$ *and* $(\gamma_c = -1/3000, \gamma_p = -1/3000)$*:* $k_u(a, w = \frac{1}{2}, \gamma_c = 500, \gamma_p = 500) = k_e(a, \gamma_c = 500, \gamma_p = 500) = 953$ *and* $k_u(a, w = \frac{1}{2}, \gamma_c = -1/3000, \gamma_p = -1/3000) = k_e(a, \gamma_c = -1/3000, \gamma_p = -1/3000) = 957$.

5.1.4 The case of homogeneous committee with complete ignorance

Suppose now the agents will not know the other players´ preferences at time of the vote. Under the this complete ignorance scenario, the best that the homoegeneous proposers can do is to make a list with his k-top alternatives, from which the chooser makes a final choice. Thus, the equilibrium outcome will be the chooser best alternative out of the k-top alternatives of the proposers. Notice that the symmetry between this characterization with the one under complete information. To see it, replace in the characterization the word "chooser" by "proposers", "proposers" by "chooser" and "k-top" by "a-k+1-top" and we have the characterization under the complete information scenario.

We can use this symmetry to characterize the distributions functions of R_p and R_c. Now, R_p has a distribution similar to the distribution of R_c under complete information scenario and, of courser, the same happen with R_c. Thus, R_p has the same distribution of a discrete random variable uniformly distributed over $\{1, 2, \ldots, k\}$ and R_c has the same distribution of the smallest element of a random sample with size $s = k$ drawn without replacement from a population $D = \{1, 2, \ldots, a\}$.

Suppose also that $u_p(R_p) = -r_p$ and $u_2(R_c) = -r_c$. The equations below give the formulas the proposer and chooser´s expected utilities:

$E(u(R_p)|a, k) = -\frac{k+1}{2}$

$E(u(R_c)|k, a) = -\frac{a+1}{k+1}$

Proposition 17 *Let* $\gamma_p = 0, \gamma_c = 0$ *and* $w = \frac{1}{2}$*:*
$k_u(a, w = \frac{1}{2}, \gamma_c = 0, \gamma_p = 0) = k_e(a, \gamma_c = 0, \gamma_p = 0) = a - \left\lfloor a + \frac{5}{2} - \sqrt{2a + \frac{9}{4}} \right\rfloor + 1 \leq \frac{(a+1)}{2}$.

Example 18 *Suppose that* $a = 1000$, $\gamma_p = 0$*,* $\gamma_c = 0$ *and* $w = \frac{1}{2}$. *Applying Proposition 11, the optimal k is equal to* $k_u(a, w = \frac{1}{2}, \gamma_c = 0, \gamma_p = 0) = a - \lfloor a + (5/2) - \sqrt[2]{(2a + 9/4)} \rfloor + 1 = 1000 - 957 + 1 = 44.0$*. Under* $k = 44$*, the proposers´ expected utilities are equal to* $-(44 + 1)/2 = -22.5$ *and the chooser's expected utility is equal to* $(1000 + 1)/(44 + 1) = -22.244$*.*

5.2 The case of several proposers: Polarized Proposers Model

In this subsection, we consider the Polarized Proposers Model, odd number of proposers and assume that agents´ preferences are random draw from an uniform distribution over the domain of preferences. We also assume that the tie breaking criterion coincides with the majoritarian group´s preferences over the set of candidates.

5.2.1 Distribution of the ranking of equilibrium outcome .

Proposition 18 *For any pair of (v,k), we have that the rankings of the strong Nash equilibrium outcome according to players preferences have the following distribution:*

1) If $m \geq q_k > n - m$ then:	
$F_1(r_1 = x \vert a,k) = \begin{cases} \sum_{j=1}^{x} \frac{\binom{a-j}{a-k}}{\binom{a}{a-k+1}} & \textit{if} \quad x \in \{1,\ldots,k\} \\ 1 & \textit{otherwise} \end{cases}$	$E(R_1 \vert a,k) = \frac{(a+1)}{(a-k+2)}$ $Var(R_1 \vert a,k,v) = \frac{(a-k+1)(a+1)(k-1)}{(a-k+2)^2(a-k+3)}$
$F_2(r_2 = x \vert a,k) = \begin{cases} \sum_{j=a-k+1}^{x} \frac{\binom{j-1}{a-k}}{\binom{a}{a-k+1}} & \textit{if} \quad x \in \{a-k+1,\ldots,a\} \\ 1 & \text{otherwise} \end{cases}$	$E(R_2 \vert a,k) = \frac{(a+1)(a-k+1)}{(a-k+2)}$ $Var(R_2 \vert a,k) = \frac{(a-k+1)(a+1)(k-1)}{(a-k+2)^2(a-k+3)}$
$F_c(r_c = x \vert k,a) = \begin{cases} \frac{x}{a-k+1} & \textit{if} \quad x \in \{1,\ldots,a-k+1\} \\ 1 & \text{otherwise} \end{cases}$	$E(R_c \vert a,k) = \frac{(a-k+2)}{2}$ $Var(R_c \vert k,a) = \frac{(a-k)(a-k+2)}{12}$

2) If $q_k > m \geq q_1 > n-m$ then:	
$F_1(r_1 = x \mid k,a) = \begin{cases} \frac{x}{k} & if \quad x \in \{1,\ldots,k\} \\ 1 & \text{otherwise} \end{cases}$	$E(R_1 \mid a,k) = \frac{k+1}{2}$ $Var(R_1 \mid a,k) = \frac{(k-1)(k+1)}{12}$
$F_2(r_2 = x \mid a,k) = \begin{cases} \frac{x}{k} & if \quad x \in \{a-k+1,\ldots,a\} \\ 0 \quad if & \text{otherwise} \end{cases}$	$E(R_2 \mid a,k) = \frac{(2a-k+1)}{2}$ $Var(R_2 \mid a,k) = \frac{(k-1)(k+1)}{12}$
$F_c(r_c = x \mid a,k) = \begin{cases} \sum_{j=1}^{x} \frac{\binom{a-j}{k-1}}{\binom{a}{k}} & if \quad x \in \{1,\ldots,a-k+1\} \\ 1 & \text{otherwise} \end{cases}$	$E(R_c \mid a,k) = \frac{(a+1)}{(k+1)}$ $Var(R_c \mid a,k) = \frac{k(a+1)(a-k)}{(k+1)^2(k+2)}$

3) If $q_k > m > n-m \geq q_1$ then:	
$F_1(r_1 = x \mid k,a) = \begin{cases} \frac{x}{k} & if \quad x \in \{1,\ldots,k\} \\ 1 & \text{otherwise} \end{cases}$	$E(R_1 \mid a,k) = \frac{a+1}{2}$ $Var(R_1 \mid a,k) = \frac{(a+1)(a-1)}{12}$
$F_2(r_2 = x \mid a,k) = \begin{cases} \frac{x}{k} & if \quad x \in \{a-k+1,\ldots,a\} \\ 0 \quad if & \text{otherwise} \end{cases}$	$E(R_2 \mid a,k) = \frac{a+1}{2}$ $Var(R_2 \mid a,k) = \frac{(a+1)(a-1)}{12}$
$F_c(r_c = x \mid a,k) = \begin{cases} 1 & if \quad x \in \{1,\ldots,a\} \\ 0 & \text{otherwise} \end{cases}$	$E(R_c \mid a,k) = 1$ $Var(R_c \mid a,k) = 0$

Corollary 6 *Consider any number of candidates a and any (k,v) such that* $m \geq q_k > n - m$*:*

1) $E(R_2) > E(R_c|k,a) > E(R_1|k,a)$ *for every* $k < a + 2 - \sqrt{2a+2}$*;*

2) $E(R_2) > E(R_c|k,a) = E(R_1|k,a)$ *if* $k = a + 2 - \sqrt{2a+2}$ *is an integer number;*

3) $E(R_2) > E(R_1|k,a) > E(R_c|k,a)$ *for every* $k > a + 2 - \sqrt{2a+2}$*.*

Corollary 7 *Consider any number of candidates a and any (k,v) such that* $q_k > m \geq q_1 > n - m$*:*

1) $E(R_2) > E(R_c|k, a) > E(R_1|k, a)$ *for every* $k < \sqrt{2a+2} - 1$;

2) $E(R_2) > E(R_c|k, a) = E(R_1|k, a)$ *if* $k = \sqrt{2a+2} - 1$ *is an integer number*;

3) $E(R_2) > E(R_1|k, a) > E(R_c|k, a)$ *for every* $k > \sqrt{2a+2} - 1$.

Corollary 8 *For any utility function* $u(\cdot)$ *and for any (k,v) such that* $q_k > m > n - m \geq q_1$*, we have that:*

$E(R_2) = E(R_1) > E(R_2)$ *for every* (k, v);

5.2.2 Proposer or chooser: which one would you like to be?

Example 19 *Consider* $a = 7, n = 5,\ m = 4$. *Suppose that the proposers use 2-rule for 3 names and that agent i has the following decreasing and concave utility function* $u(r) = -r$. *Proposer or Chooser: which position would give a higher payoff to agent i? Before answer this question, let us compute the expected returns and risks face by the proposer and chooser. Notice first that, given k=3 and v=2, we have that* $q_k > m \geq q_1 > n - m$. *Hence, applying Proposition 18, we have that:*

$E(u(R_1)|a = 7, k = 3, v = 2) = -2$; $E(u(R_2)|a = 7, k = 3, v = 2) = -6$ *and* $E(u(R_c)|a = 7, k = 3, v = 2) = -2$;

Thus, the agent i is indifferent between to be the a proposer of the majoritarian group and the chooser.

However, if agent i was risk averse, so he cares about the risks of being in each position. Suppose that agent i has the following decreasing and concave utility function $u(r) = -e^{\gamma r}$ *where* $\gamma > 0$. *Let us assume that* $\gamma = 0.2$. *So, his expected utility in each position is:* $E(u(R_1)|a = 7, k = 3, v = 2) = \sum_{j=1}^{3} \frac{-e^{0.2j}}{3} = -1.5118$, $E(u(R_2)|a = 7, k = 3, v = 2) = \sum_{j=5}^{7} \frac{-e^{0.2j}}{3} = -3.3645$ *and* $E(u(R_c)|a = 17, k = 12) = \sum_{j=1}^{5} \frac{-e^{0.2j}\binom{7-j}{3-1}}{\binom{7}{3}} = -1.5305$

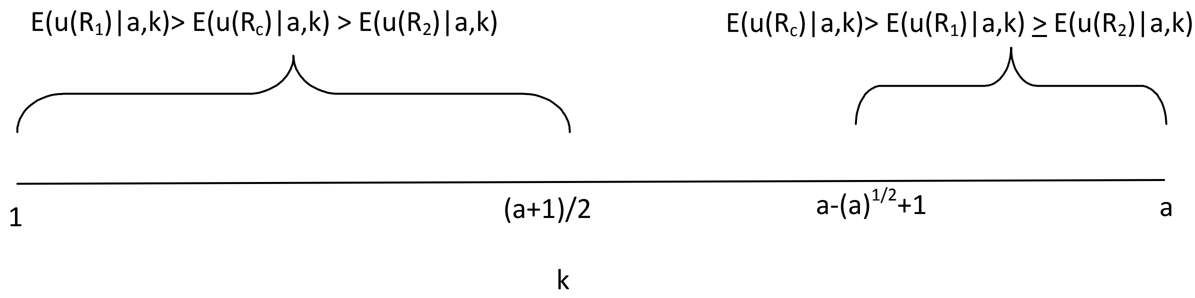
Thus, now, agent i prefers to be a proposer of the majoritarian group. Suppose now that proposers use 1-rule for 3 names. Given k=3 and v=1 we have that $q_k > m > n - m \geq q_1$.

Thus, applying Proposition 18, we have $E(u(R_1)|a = 7, k = 3, v = 1) = \sum_{j=1}^{7} \frac{-e^{0.2j}}{7} = -2.4078$, $E(u(R_2)|a = 7, k = 3, v = 2) = \sum_{j=1}^{7} \frac{-e^{0.2j}}{7} = -2.4078$ *and* $E(u(R_c)|a = 7, k = 3, v = 1) = -e^{0.2} = -1.2214$. *Hence, when v=1 and k=3, agent i prefers to be the chooser.*

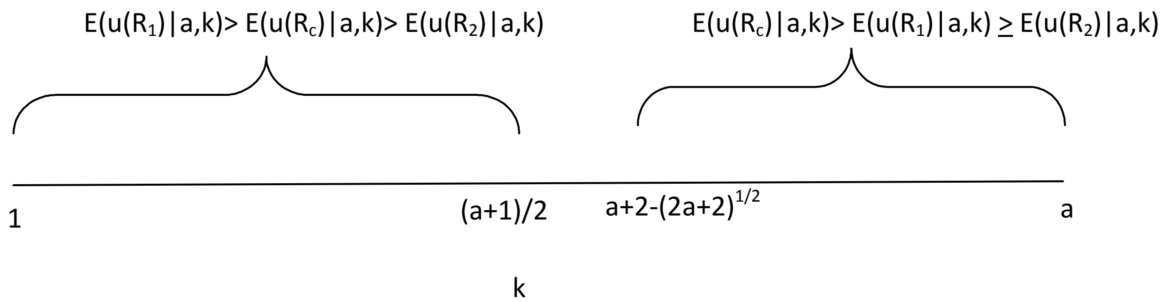
Notice also if agent i had $u(r) = -r$, *i.e he were risk neutral. He would still prefer to be the chooser given that* $E(u(R_1)|a = 7, k = 3, v = 1) = -4$; $E(u(R_2)|a = 7, k = 3, v = 1) = -4$ *and* $E(u(R_c)|a = 7, k = 3, v = 1) = -1$.

Proposition 19 *Consider any number of candidates a and any (k,v) such that $m \geq q_k > n - m$:*

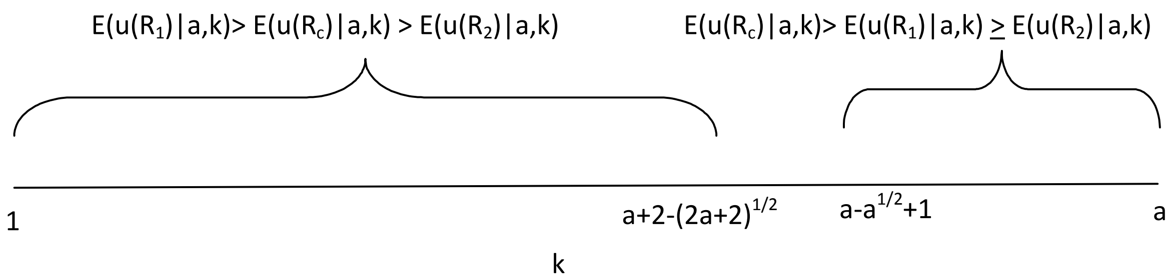
1) For any decreasing utility function, we have:



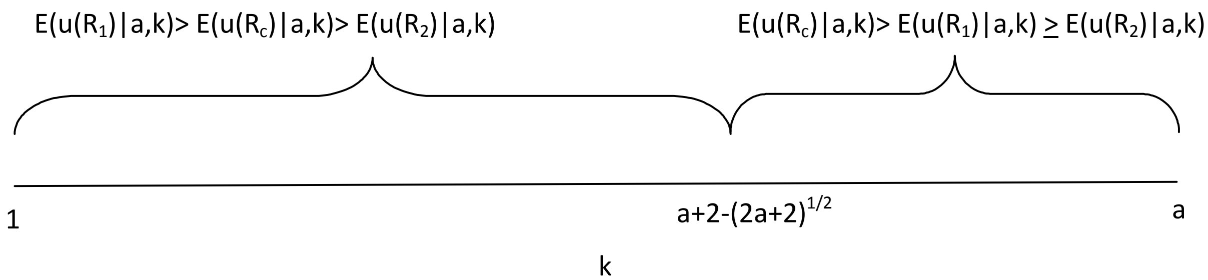
2) For any concave and decreasing utility function, we have:



3) For any convex and decreasing utility function, we have:

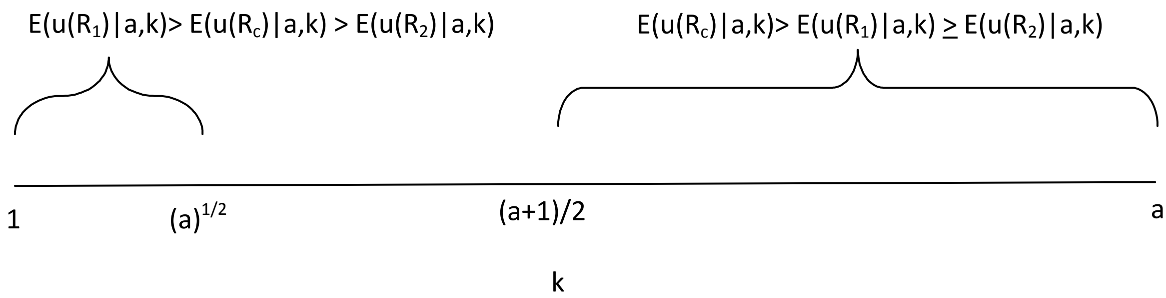


4) For any linear and decreasing utility function, we have:

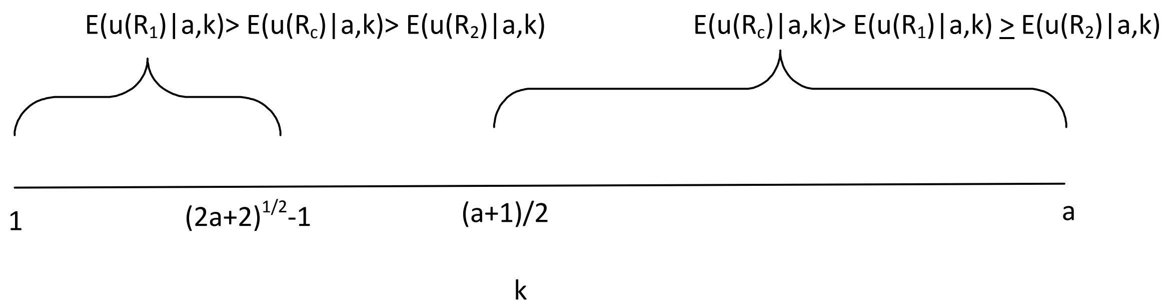


Proposition 20 *Consider any number of candidates a and any (k,v) such that $q_k > m \geq q_1 > n - m$:*

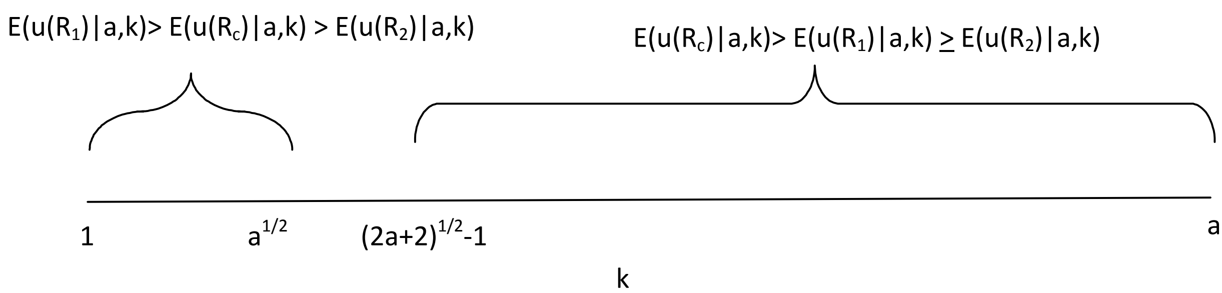
1) For any decreasing utility function, we have:



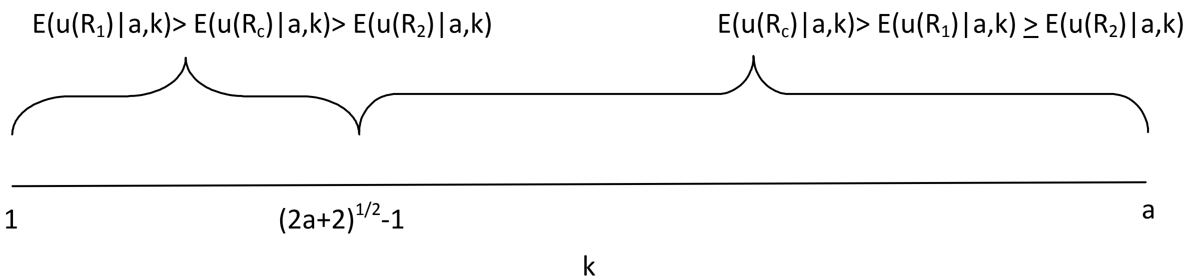
2) For any concave and decreasing utility function, we have:



3) For any convex and decreasing utility function, we have:

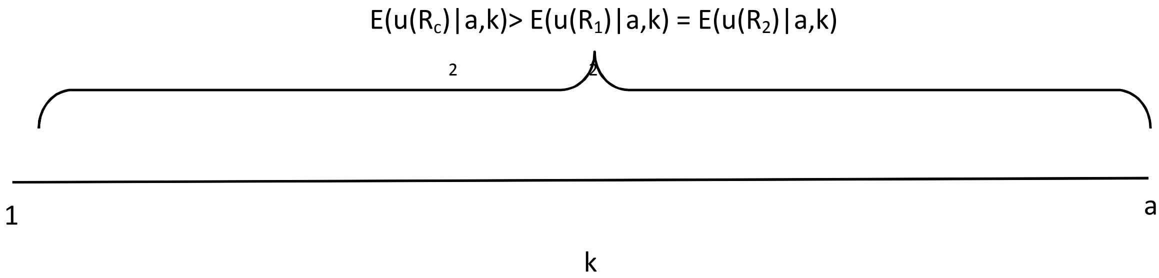


4) For any linear and decreasing utility function, we have:



Proposition 21 *Consider any number of candidates a and any (k,v) such that* $q_k > m > n - m \geq q_1$*:*

For any linear and decreasing utility function, we have:



5.2.3 Welfare analysis: The optimal k

Definition 16 *Consider the Polarized Proposers Model. A pair* $(k, v) \in \{1, ..., a\} \times \{1, ..., k\}$ *satisfies the egalitarian criterion if* $|\frac{m}{n}E(u_1(R_1)|k, v, a) + \frac{n-m}{n}E(u_2(R_2)|k, v, a) - E(u_c(R_c)|k, v, a)|$
$\leq |\frac{m}{n}E(u_1(R_1)|k\acute{}, v\acute{}, a) + \frac{n-m}{n}E(u_2(R_2)|k\acute{}, v\acute{}, a) - E(u_c(R_c)|k\acute{}, v\acute{}a)|$ *for every* $(k\acute{}, v\acute{}) \in \{1, ..., a\} \times \{1, ..., k\acute{}\}$*. We denote* (k_e, v_e) *the largest* $(k, v) \in \{1, ..., a\} \times \{1, ..., k\}$ *that satisfies the egalitarian criterion.*

Proposition 22 *In the domain of all pairs* (k, v) *such that* $m \geq q_k > n - m$*, we have that:*

1) $E(R_c|k, a) > \frac{m}{n}E(R_1|k, v, a) + \frac{n-m}{n}E(R_2|k, v, a)$ *for every* $k < \tau_1$*;*

2) $\frac{m}{n}E(R_1|k, v, a) + \frac{n-m}{n}E(R_2|k, v, a) = E(R_c|k, a)$ *if* $k = \tau_1$ *is an integer number;*

3) $\frac{m}{n}E(R_1|k, v, a) + \frac{n-m}{n}E(R_2|k, v, a) < E(R_c|k, a)$ *for every* $k > \tau_1$*.*

where $\tau_1 = \frac{m}{n}\left(\frac{n}{m} + (a+1) - \sqrt{\frac{n}{m}(2 - \frac{n}{m}) + (2a+1) + a^2(\frac{n}{m} - 1)^2}\right)$

Proof. Given that $m \geq q_k > n - m$, by Proposition 18, we have that $E(R_1|k, v, a) = \frac{(a+1)}{a-k+2}$,
$E(R_2|k, v, a) = -\frac{(a+1)(a-k+1)}{a-k+2}$ and $E(R_c|k, v, a) = (\frac{a-k+2}{2})$. Notice that:
$|\frac{m}{n}E(R_1|k, v, a) + \frac{n-m}{n}E(R_2|k, v, a) - E(R_c|k, v, a)|$ is single dipped and reaches the minimum when $k = \tau_1$.When $k = \tau_1$, we have that $|\frac{m}{n}E(R_1|k, v, a) + \frac{n-m}{n}E(R_2|k, v, a) - E(R_c|k, v, a)| = 0$. ■

Proposition 23 *In the domain of all pairs (k, v) such that $q_k > m \geq q_1 > n - m$, we have that:*

1) $E(R_c|k, a) > \frac{m}{n}E(R_1|k, v, a) + \frac{n-m}{n}E(R_2|k, v, a)$ for every $k < \tau_2$;

2) $\frac{m}{n}E(R_1|k, v, a) + \frac{n-m}{n}E(R_2|k, v, a) = E(R_c|k, a)$ if $k = \tau_2$ is an integer number;

3) $\frac{m}{n}E(R_1|k, v, a) + \frac{n-m}{n}E(R_2|k, v, a) < E(R_c|k, a)$ for every $k > \tau_2$.

where $\tau_2 = \frac{\frac{m}{n}}{2\frac{m}{n}-1}\left((a-1) - a\frac{n}{m} + \sqrt{\frac{n}{m}(2-\frac{n}{m}) + (2a+1) + a^2(\frac{n}{m}-1)^2}\right)$

Proof. Given that $q_k > m \geq q_1 > n - m$, by Proposition 18, we have that $E(R_1|k, v, a) = \frac{k+1}{2}$, $E(R_2|k, v, a) = \frac{2a-k+1}{2}$ and $E(R_c|k, v, a) = \frac{a+1}{k+1}$.
Notice that
$|\frac{m}{n}E(R_1|k, v, a) + \frac{n-m}{n}E(R_2|k, v, a) - E(R_c|k, v, a)|$ is single dipped and reaches the minimum when $k = \tau_2$. When $k = \tau_2$, we have that $|\frac{m}{n}E(R_1|k, v, a) + \frac{n-m}{n}E(R_2|k, v, a) - E(R_c|k, v, a)| = 0$. ■

Corollary 9 *Consider the Polarized Proposers Model and suppose that $\gamma_1 = \gamma_2 = \gamma_c = 0$: $(k_e, v_e) \in \{(\lfloor\tau_1\rfloor, \lfloor\tau_1\rfloor)), (\lceil\tau_1\rceil, \lceil\tau_1\rceil), (\lfloor\tau_2\rfloor, v_2(\lfloor\tau_2\rfloor)), (\lceil\tau_2\rceil, v_2(\lceil\tau_2\rceil))\}$.*
where
$\tau_1 = \frac{m}{n}\left(\frac{n}{m} + (a+1) - \sqrt{\frac{n}{m}(2-\frac{n}{m}) + (2a+1) + a^2(\frac{n}{m}-1)^2}\right)$;
$\tau_2 = \frac{\frac{m}{n}}{2\frac{m}{n}-1}\left((a-1) - a\frac{n}{m} + \sqrt{\frac{n}{m}(2-\frac{n}{m}) + (2a+1) + a^2(\frac{n}{m}-1)^2}\right)$;
$v_2(k) = max\{\{v\epsilon\{1, ..., k\}|q_k > m \geq q_1 > n - m\} \cup \{1\}\}$.

Example 19 shows that the egalitarian k under the Polarized Proposers Model is not larger than egalitarian k under homogeneous committee. Moreover, the egalitarian k is lower as more polarized is the committee. The example also shows that, different from the homogenous committee case, the egalitarian k does not maximize the weighted the sum of expected utilities.

Example 20 *Let $a = 10, n = 7$, $m = 5$ and $\gamma_1 = \gamma_2 = \gamma_c = 0$. Applying Corollary 9, we have that $\tau_1 = 4.4633$, $\tau_2 = 1.919$,$v_2(\lfloor\tau_2\rfloor) = 1$ and $v_2(\lceil\tau_2\rceil) = 1$. Hence, $(k_e, v_e) \in \{(4, 4), (5, 5), (1, 1), (2, 1))\}$. In Table 1, we can see that $(k_e, v_e) = (2, 1)$. Now, assume that $m = 6$. Again, applying Corollary 9, we have that $\tau_1 = 6.1643$, $\tau_2 = 2.7699$, $v_2(\lfloor\tau_2\rfloor) = 1$*

and $v_2(\lceil\tau_2\rceil) = 1$. *Hence,* $(k_e, v_e) \in \{(6,6),(7,7),(2,1),(3,1))\}$. *The reader can check in Table 2 that* $(k_e, v_e) = (6,6)$. *Notice that as the size of the majority increases from 5 to 6, the egalitarian k increases from 2 to 6. In the homogeneous commitee case, the optimal k would be* 7. *Notice also the chooser´s payoff increases as the parameter* v *and the size of the majority* m *decrease and k increases.*

Table 1: Agents´ Expected Utilities for different values of k and v under a=10, n=7 and m=5 (Polarized Proposers Model)

k	v	q_1	q_k	Condition	$E(u_1(r_1))$	$E(u_2(r_2))$	$E(u_p)$ [1]	$E(u_c(r_c))$	$\lvert E(u_p)-E(u_c(R_c))\rvert$	$E(u_p)+E(u_c(R_c))$
1	1	4	4	m≥qk>n-m	-1,00	-10,00	-3,57	-5,50	1,93	-9,07
2	1	3	6	qk>m≥q1>n-m	-1,50	-9,50	-3,79	-3,67	0,12	-7,45
2	2	4	4	m≥qk>n-m	-1,10	-9,90	-3,61	-5,00	1,39	-8,61
3	1	2	6	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
3	2	3	5	m≥qk>n-m	-1,22	-9,78	-3,67	-4,50	0,83	-8,17
3	3	4	4	m≥qk>n-m	-1,22	-9,78	-3,67	-4,50	0,83	-8,17
4	1	2	7	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
4	2	3	6	qk>m≥q1>n-m	-2,50	-8,50	-4,21	-2,20	2,01	-6,41
4	3	4	5	m≥qk>n-m	-1,38	-9,63	-3,73	-4,00	0,27	-7,73
4	4	4	4	m≥qk>n-m	-1,38	-9,63	-3,73	-4,00	0,27	-7,73
5	1	2	7	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
5	2	3	6	qk>m≥q1>n-m	-3,00	-8,00	-4,43	-1,83	2,60	-6,26
5	3	3	5	m≥qk>n-m	-1,57	-9,43	-3,82	-3,50	0,32	-7,32
5	4	4	5	m≥qk>n-m	-1,57	-9,43	-3,82	-3,50	0,32	-7,32
5	5	4	4	m≥qk>n-m	-1,57	-9,43	-3,82	-3,50	0,32	-7,32
6	1	2	7	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
6	2	2	6	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
6	3	3	6	qk>m≥q1>n-m	-3,50	-7,50	-4,64	-1,57	3,07	-6,21
6	4	3	5	m≥qk>n-m	-1,83	-9,17	-3,93	-3,00	0,93	-6,93
6	5	4	5	m≥qk>n-m	-1,83	-9,17	-3,93	-3,00	0,93	-6,93
6	6	4	4	m≥qk>n-m	-1,83	-9,17	-3,93	-3,00	0,93	-6,93
7	1	1	7	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
7	2	2	7	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
7	3	3	6	qk>m≥q1>n-m	-4,00	-7,00	-4,86	-1,38	3,48	-6,23
7	4	3	6	qk>m≥q1>n-m	-4,00	-7,00	-4,86	-1,38	3,48	-6,23
7	5	3	5	m≥qk>n-m	-2,20	-8,80	-4,09	-2,50	1,59	-6,59
7	6	4	5	m≥qk>n-m	-2,20	-8,80	-4,09	-2,50	1,59	-6,59
7	7	4	4	m≥qk>n-m	-2,20	-8,80	-4,09	-2,50	1,59	-6,59
8	2	2	7	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
8	3	2	6	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
8	4	3	6	qk>m≥q1>n-m	-4,50	-6,50	-5,07	-1,22	3,85	-6,29
8	5	3	5	m≥qk>n-m	-2,75	-8,25	-4,32	-2,00	2,32	-6,32
8	6	4	5	m≥qk>n-m	-2,75	-8,25	-4,32	-2,00	2,32	-6,32
8	7	4	5	m≥qk>n-m	-2,75	-8,25	-4,32	-2,00	2,32	-6,32
8	8	4	4	m≥qk>n-m	-2,75	-8,25	-4,32	-2,00	2,32	-6,32
9	2	2	7	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
9	3	2	6	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
9	4	3	6	qk>m≥q1>n-m	-5,00	-6,00	-5,29	-1,10	4,19	-6,39
9	5	3	6	qk>m≥q1>n-m	-5,00	-6,00	-5,29	-1,10	4,19	-6,39
9	6	3	5	m≥qk>n-m	-3,67	-7,33	-4,71	-1,50	3,21	-6,21
9	7	4	5	m≥qk>n-m	-3,67	-7,33	-4,71	-1,50	3,21	-6,21
9	8	4	5	m≥qk>n-m	-3,67	-7,33	-4,71	-1,50	3,21	-6,21
9	9	4	4	m≥qk>n-m	-3,67	-7,33	-4,71	-1,50	3,21	-6,21
10	2	2	7	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	3	2	7	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	4	3	6	qk>m≥q1>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	5	3	6	qk>m≥q1>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	6	3	5	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	7	3	5	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	8	4	5	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	9	4	5	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	10	4	4	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50

1: E(up)=(n/m)E(u1(R1))+(1-m/n)E(u2(R2))

Table 2: Agents´ Expected Utilities for different values of k and v under a=10, n=7 and m=6 (Polarized Proposers Model)

k	v	q_1	q_k	Condition	$E(u_1(r_1))$	$E(u_2(r_2))$	$E(u_p)$ [1]	$E(u_c(r_c))$	$\lvert E(u_p)-E(u_c(R_c))\rvert$	$E(u_p)+E(u_c(R_c))$
1	1	4	4	m≥qk>n-m	-1,00	-10,00	-2,29	-5,50	3,21	-7,79
2	1	3	6	m≥qk>n-m	-1,10	-9,90	-2,36	-5,00	2,64	-7,36
2	2	4	4	m≥qk>n-m	-1,10	-9,90	-2,36	-5,00	2,64	-7,36
3	1	2	6	m≥qk>n-m	-1,22	-9,78	-2,44	-4,50	2,06	-6,94
3	2	3	5	m≥qk>n-m	-1,22	-9,78	-2,44	-4,50	2,06	-6,94
3	3	4	4	m≥qk>n-m	-1,22	-9,78	-2,44	-4,50	2,06	-6,94
4	1	2	7	qk>m≥q1>n-m	-2,50	-8,50	-3,36	-2,20	1,16	-5,56
4	2	3	6	m≥qk>n-m	-1,38	-9,63	-2,55	-4,00	1,45	-6,55
4	3	4	5	m≥qk>n-m	-1,38	-9,63	-2,55	-4,00	1,45	-6,55
4	4	4	4	m≥qk>n-m	-1,38	-9,63	-2,55	-4,00	1,45	-6,55
5	1	2	7	qk>m≥q1>n-m	-3,00	-8,00	-3,71	-1,83	1,88	-5,55
5	2	3	6	m≥qk>n-m	-1,57	-9,43	-2,69	-3,50	0,81	-6,19
5	3	3	5	m≥qk>n-m	-1,57	-9,43	-2,69	-3,50	0,81	-6,19
5	4	4	5	m≥qk>n-m	-1,57	-9,43	-2,69	-3,50	0,81	-6,19
5	5	4	4	m≥qk>n-m	-1,57	-9,43	-2,69	-3,50	0,81	-6,19
6	1	2	7	qk>m≥q1>n-m	-3,50	-7,50	-4,07	-1,57	2,50	-5,64
6	2	2	6	m≥qk>n-m	-1,83	-9,17	-2,88	-3,00	0,12	-5,88
6	3	3	6	m≥qk>n-m	-1,83	-9,17	-2,88	-3,00	0,12	-5,88
6	4	3	5	m≥qk>n-m	-1,83	-9,17	-2,88	-3,00	0,12	-5,88
6	5	4	5	m≥qk>n-m	-1,83	-9,17	-2,88	-3,00	0,12	-5,88
6	6	4	4	m≥qk>n-m	-1,83	-9,17	-2,88	-3,00	0,12	-5,88
7	1	1	7	qk>m>n-m≥q1	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
7	2	2	7	qk>m≥q1>n-m	-4,00	-7,00	-4,43	-1,38	3,05	-5,80
7	3	3	6	m≥qk>n-m	-2,20	-8,80	-3,14	-2,50	0,64	-5,64
7	4	3	6	m≥qk>n-m	-2,20	-8,80	-3,14	-2,50	0,64	-5,64
7	5	3	5	m≥qk>n-m	-2,20	-8,80	-3,14	-2,50	0,64	-5,64
7	6	4	5	m≥qk>n-m	-2,20	-8,80	-3,14	-2,50	0,64	-5,64
7	7	4	4	m≥qk>n-m	-2,20	-8,80	-3,14	-2,50	0,64	-5,64
8	2	2	7	qk>m≥q1>n-m	-4,50	-6,50	-4,79	-1,22	3,56	-6,01
8	3	2	6	m≥qk>n-m	-2,75	-8,25	-3,54	-2,00	1,54	-5,54
8	4	3	6	m≥qk>n-m	-2,75	-8,25	-3,54	-2,00	1,54	-5,54
8	5	3	5	m≥qk>n-m	-2,75	-8,25	-3,54	-2,00	1,54	-5,54
8	6	4	5	m≥qk>n-m	-2,75	-8,25	-3,54	-2,00	1,54	-5,54
8	7	4	5	m≥qk>n-m	-2,75	-8,25	-3,54	-2,00	1,54	-5,54
8	8	4	4	m≥qk>n-m	-2,75	-8,25	-3,54	-2,00	1,54	-5,54
9	2	2	7	qk>m≥q1>n-m	-5,00	-6,00	-5,14	-1,10	4,04	-6,24
9	3	2	6	m≥qk>n-m	-3,67	-7,33	-4,19	-1,50	2,69	-5,69
9	4	3	6	m≥qk>n-m	-3,67	-7,33	-4,19	-1,50	2,69	-5,69
9	5	3	6	m≥qk>n-m	-3,67	-7,33	-4,19	-1,50	2,69	-5,69
9	6	3	5	m≥qk>n-m	-3,67	-7,33	-4,19	-1,50	2,69	-5,69
9	7	4	5	m≥qk>n-m	-3,67	-7,33	-4,19	-1,50	2,69	-5,69
9	8	4	5	m≥qk>n-m	-3,67	-7,33	-4,19	-1,50	2,69	-5,69
9	9	4	4	m≥qk>n-m	-3,67	-7,33	-4,19	-1,50	2,69	-5,69
10	2	2	7	qk>m≥q1>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	3	2	7	qk>m≥q1>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	4	3	6	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	5	3	6	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	6	3	5	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	7	3	5	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	8	4	5	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	9	4	5	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50
10	10	4	4	m≥qk>n-m	-5,50	-5,50	-5,50	-1,00	4,50	-6,50

1: E(up)=(n/m)E(u1(R1))+(1-m/n)E(u2(R2))

6 Concluding Remarks

We generalize Barberà and Coelho (2010) characterization of the strong Nash equilibrium outcomes of the game induced by the rule of k names. We provide necessary and sufficient conditions of a candidate to be a strong Nash equilibrium outcome of the Constrained Chooser Game when the screening rule for k names belongs to a family of $v-$votes screening rule for k names. A screening rule for k names is a $v-$votes screening rule for k names if it can be described as follows: Each proposer votes for v candidates and the list has the names of the k most voted candidates, with a tie breaking rule when needed. Our characterization is based on two parameters of the screening rule for k names. These two parameters measure in two different dimensions the size of a winning coalition. The first parameter, q_k, is the size of the smaller coalition that is able to impose all the k names of the list. The second parameter, q_1, is the size of the smaller coalition that is able to impose at least one name in the list. We show that the chooser´s payoff is a increasing function on the size of list and level of polarization of the proposers preferences and it is decreasing on parameter v of the screening rule.

Our second perspective builds on what we learn about equilibria, but address a more aggregate question: what is the performance of each of these rules "in average"? More specifically, we study the agents´s payoff distributions from applying different v-rules of k names. Based on these payoff distributions, we characterize the optimal k and the screening rule according to utilitarian and egalitarian criteria.

While we do not provide in this paper an endogenous explanation for the choice of k, it turns out that the optimal k is a function of players´ degree of risk aversion. If there is only one proposer and the agents are risk neutral, the k that minimizes the sum of expected utility for the chooser and for the proposer is also the one that tends to equalize these two values.

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Appendix

Proof of Proposition 1. Take any coalition $Q \subseteq N$ with $\#|Q| = q$ and any subset of candidates $B \subseteq A$ with $\#|B| = k$. Suppose that the members of coalition Q coordinates their votes in order to elect B. The worst scenario is the one where the complementary coalition $N\backslash Q$ vote all together for some $a \in A/B$, it implies that a will receive $n-q$ votes. Given this worst scenario, the best coalition Q can do to ensure the selection of B is to spread equally, as much as possible, their $v.q$ votes among the k candidates in B, given this strategy, the number of votes that any candidate in B receives is $\lfloor \frac{qv}{k} \rfloor$ or $\frac{qv}{k}$. Thus, the set B will be elected if $\lfloor \frac{qv}{k} \rfloor > n-q$. By definition, q_k is the minimum q that the following inequality holds: $\lfloor \frac{qv}{k} \rfloor > n-q$. It implies that $q_k = \lceil \frac{kn}{(k+v)} \rceil + \mathcal{I}(\lfloor \frac{v\lceil \frac{kn}{(k+v)} \rceil}{k} \rfloor \leq n - \lceil \frac{kn}{(k+v)} \rceil)$, where $\mathcal{I}$ denotes the indicator function. Thus the first part of the proposition is established

Take any coalition $Q \subseteq N$ with $\#|Q| = q$ and any candidate $a \in A$. Suppose that the members of coalition Q coordinates their votes in order to ensure a as one of ks selected names. The worst scenario is the one where the complementary coalition $N\backslash Q$ distribute equally ,as much as possible, their $v(n-q)$ among others k candidates,some $B \subseteq A/\{a\}$ with $\#|B| = k$, it implies that at least one candidate in B receives $\lfloor \frac{(n-q)v}{k} \rfloor$ votes. Given this worst scenario, the best response of coalition Q to ensure the inclusion of a is to all together vote for a, given this strategy, a candidate a will receives q votes. Thus, candidate a will be one k-listed names if $q > \lfloor \frac{(n-q)v}{k} \rfloor$. By definition, q_1 is the minimum q that this inequality holds. It implies that $q_1 = \lceil \frac{vn}{(k+v)} \rceil + \mathcal{I}(\frac{vn}{(k+v)} = \lceil \frac{vn}{(k+v)} \rceil)$, where $\mathcal{I}$ denotes the indicator function. ■

Proof of Proposition 2. Suppose that candidate x is the outcome of a strong Nash equilibrium of the Constrained Chooser Game. In any strong Nash equilibrium where x is the outcome, the screened set is such that x is the best candidate in this set according to the chooser's preferences. So, it implies that x is a chooser's $(\#\mathbf{A}-k+1)$-top candidate. Take any candidate among those that are chooser's $(\#A-k+1)$-top candidates and denote him by y and Y any list with k names where y is the chooser best candidate in Y, notice that y is not considered better than x by any coalition with at least $q_k(Y)$ candidates. Otherwise, this coalition could impose Y, preventing x being elected. So, If y is a chooser's $(\#A-k+1)$-top candidate then $\#\{i \in N | y \succ_i x\} < q_k(Y)$ for any $Y \in A_k$ such that y is the chooser best candidate in Y. Now, we need to show that if a candidate y is the chooser best candidate then $\#\{i \in N | y \succ_i x\} < q_1(y)$. Suppose, by contradiction, that it is not true then $\#\{i \in N | y \succ_i x\} \geq q_1(y)$. Let denote $C_1 \equiv \{i \in N | y \succ_i x\}$,so $\#C_1 \geq q_1(y)$. This is a contradiction, because the coalition of proposers in C_1 would be able to impose (since $\#C_1 \geq q_1(y)$) the inclusion of y in the list and the chooser would select it instead of x. Hence, if y the chooser's best candidate, we have that $\#\{i \in N | y \succ_i x\} < q_1(y)$. ■

Proof of Propostion 3. Suppose that a candidate x is one of the chooser´s $(\#A - k + 1)$-top candidates, $Y \in A_k$ such that x is the chooser best candidate in X and $q_k(X)$ proposers rank x highest. Now let us show that there is a strategy profile that sustains x as a strong Nash equilibrium outcome. Let $C \subset N$ be the set of proposers that rank highest x, so $\#C \geq q_k(X)$. By defintion of $q_k(Y)$, there exists $m_C \in M^C$ such that for every profile of the complementary coalition $m_{\mathbf{N}\backslash C} \in M^{\mathbf{N}\backslash C}$ we have that $S_k(m_C, m_{\mathbf{N}\backslash C}) = X$. Consider any strategy profile, where the coalition C uses m_C. At this strategy profile X will be selected and x will be the winning candidate indepedently of the actions of the complementary coalition. Thus, there is no coalition of proposers that has incentives in deviating. Therefore, this strategy profile sustains x as a strong Nash equilibrium outcome. Now let us show that x is the unique strong Nash equilibrium outcome. Suppose by contradiction that there is a strategy profile that sustains $y \in A \backslash \{x\}$ as strong Nash equilibrium outcome. By Proposition 2, it implies $\#\{i \in N | x \succ_i y\} < q_k(X)$ and y is a chooser's $(\#\mathbf{A} - k + 1)$-top candidate. It is a contradiction since x is a $q_k(X)$-Condorcet winner over the set of the chooser's $(\#\mathbf{A} - k + 1)$-top candidates. ■

Proof of Proposition 4. Suppose that a candidate x is a $n - \lfloor \frac{nv}{2k} \rfloor + 1-$Condorcet winner over the set of chooser's $(\#\mathbf{A} - k + 1)$-top candidates. Now let us show that there is a strategy profile that sustains x as a strong Nash equilibrium outcome. Take any set $B \subseteq A$ with $\#|B| = k$ where x is chooser best candidate in the set (this set only exists because x is a chooser's $(\#\mathbf{A} - k + 1)$-top candidate). Consider a strategy profile, where each candidate in B receives at least $\lfloor \frac{nv}{k} \rfloor$ votes and all the candidates in $A \backslash B$ receive zero votes. Notice that the candidates in B will form the chosen list. Then, candidate x will be elected since he is the best chooser´s candidate in the chosen list. In order to change this result, the only way is to avoid the inclusion of x in the list or substituting another listed name by a candidate considered better that x by the chooser. A necessary condition to make this change would require to transfer at least $\lfloor \frac{nv}{2k} \rfloor$ votes of a candidate in B to another candidate in $A/\{B\}$. So, any coalitions with size smaller than $\lfloor \frac{nv}{2k} \rfloor$ cannot avoid the inclusion of x in the chosen list. Notice that any coalition with size higher or equal to $\lfloor \frac{nv}{2k} \rfloor$ does not have any incentive to deviate, since there is no $y \in A \backslash \{x\}$ among the chooser's $(\#\mathbf{A} - k + 1)$-top candidates such that $\#\{i \in N | y \succ_i x\} \geq \lfloor \frac{nv}{2k} \rfloor$ (recall that only the chooser's $(\#\mathbf{A} - k + 1)$-top candidates can be the chooser's best name among the candidates of a set with cardinality k). Otherwise, x would not be a $n-\lfloor \frac{nv}{2k} \rfloor+1-$Condorcet winner over the set of the chooser's $(\#\mathbf{A}-k+1)$-top candidates. Therefore, this strategy profile is a strong Nash equilibrium of the Constrained Chooser Game.

Now let us show that x is the unique strong Nash equilibrium outcome. First notice $n-\lfloor \frac{nv}{2k} \rfloor+1 \geq q_k$ (Because any coalition with size higher than $n-\lfloor \frac{nv}{2k} \rfloor+1$ can impose all the k names in the list), so x is also a q_k-Condorcet winner over the set of chooser's $(\#\mathbf{A}-k+1)$-top candidates. Suppose by contradiction that there is a strategy profile that sustains $y \in A\backslash\{x\}$ as strong Nash equilibrium outcome. By Proposition 2, it implies $\#\{i \in N | x \succ_i y\} < q_k$ and y is a chooser's $(\#\mathbf{A}-k+1)$-top candidate. It is a contradiction since x is a q_k-Condorcet winner over the set of the chooser's $(\#\mathbf{A}-k+1)$-top candidates.

■

Proof of Proposition 5. Suppose that a candidate x is the chooser´s top candidate and it is also a $q_1(x)-$Condorcet winner over the set of chooser's $(\#\mathbf{A}-k+1)$-top candidates. First let us show that there is a strategy profile that sustains x as a strong Nash equilibrium outcome. Consider the strategy profile, where all proposers votes for x. Notice that x will be in the chosen list.

Then, candidate x will be elected since he will be in the list and he is the chooser´s top candidate. The only way to change this result is to avoid the inclusion of x in the chosen list. So, any coalitions with size smaller than $n-q_1(x)$ cannot avoid the inclusion of x in the chosen list, because the complementary coalition would have size higher than $q_1(x)$. Notice that any coalition with size higher or equal to $n-q_1(x)+1$ does not have any incentive to deviate, since there is no $y \in A\backslash\{x\}$ among the chooser's $(\#\mathbf{A}-k+1)$-top candidates that is considered better than x by all proposers in the coalition (recall that only the chooser's $(\#\mathbf{A}-k+1)$-top candidates can be the chooser' best name among the candidates of a set with cardinality k). Otherwise, x would not be a $q_1(x)$-Condorcet winner over the set of the chooser's $(\#\mathbf{A}-k+1)$-top candidates. Therefore, this strategy profile is a strong Nash equilibrium of the Constrained Chooser Game.

Now let us show that x is the unique strong Nash equilibrium outcome. Suppose by contradiction that there is a strategy profile that sustains $y \in A\backslash\{x\}$ as strong Nash equilibrium outcome. By Proposition 2, it implies that y is a chooser's $(\#\mathbf{A}-k+1)$-top candidate and $\#\{i \in N | x \succ_i y\} < q_1(x)$. It is a contradiction since x is a $q_1(x)$-Condorcet winner over the set of the chooser's $(\#\mathbf{A} - k + 1)$-top candidates. ■

Proof of Proposition 6. First notice that given that chooser´s 1-top candidate is a strong Nash equilibrium outcome under a $v\acute{}-$votes screening rule for k names, it implies that any strategy profile where all proposers votes for x is a strong Nash equilibrium. Take any strategy profile where all voters vote for x, and call by $m\acute{}$. Given that it is a strong Nash equilibrium, there is no coalition of voters that can make a profitable deviation. The voters that would wish to avoid the election of x are those that prefer another chooser's $(\#\mathbf{A}-k+1)$-top candidate to x (recall that only the chooser's $(\#\mathbf{A}-k+1)$-top candidates can be the chooser' best name among the candidates of a set with cardinality k).The only way to avoid the election of x would be to avoid the inclusion of x in the chosen list. Take any chooser's $(\#\mathbf{A} - k + 1)$-top candidate and call it by y. If all the voters that prefer y to x deviate from $m\acute{}$ by do not vote for x, x would continue to have enough votes to be one name of k listed names. Otherwise, the strategy profile where all the voters vote for x would no be a strong nash equilibrium.
Now let us show that x is also a strong Nash equilibrium any $\widetilde{v}-$votes screening rule for k names where $\widetilde{v} < v\acute{}$. We need to show that there is a strategy profile that sustains x as strong Nash equilibrium outcome under $\widetilde{v}-$votes screening rule for k names.
Take any strategy profile where all voters vote for x and call this strategy by $\widetilde{m}$. So, x will be one of k listed name and it will be the elected candidate. We need to show that there is no coalition of voters that can make a profitable deviation under $m\acute{}$. Given $m\acute{}$ and $\widetilde{m}$, notice that it is more difficult to make a profitable deviation under $\widetilde{v}-$votes screening rule for k names than $v\acute{}-$votes screening rule for k names. Because, under a $\widetilde{v}-$votes screening rule for k names, any coalition of voters that would have incentive to avoid the election of x has less votes to distribute among the k candidates in order to avoid the inclusion of x in the list. Thus, given that there exists no coalition that can make a profitable deviation under $m\acute{}$, it implies that there exists no coalition that can make a profitable deviation under $\widetilde{m}$. Therefore, x is a strong Nash equilibrium outcome under $\widetilde{v}-$votes screening rule for k names. ■

Proof of Proposition 8. Let x be an outcome of a strong Nash equilibrium of the Constrained Chooser Game. By Condition 1 of Proposition 2, it implies that x is a chooser's $(\#\mathbf{A} - k + 1)$-top candidate.
Suppose that $m \geq q_k$. Since $m \geq q_k$, the voters in the majoritarian group can impose all

the names in the list . Notice that there exists no candidate among those that are chooser's $(\#A{-}k+1)$-top candidates that is considered better than x by the majoritarian group of voters. Otherwise, this group could impose a list with this candidate in which he would be the chooser´s preferred candidate, since he is also one of the chooser's $(\#A{-}k{+}1)$-top candidates, preventing x being elected. So, if y is a chooser's $(\#A{-}k+1)$-top candidate then $\#\{i \in N|y \succ_i x\} < q_k$. Therefore, the equilibrium outcome of the Constrained Chooser Game is the best alternative of the majoritarian group out of chooser´s $\#A{-}k + 1$-top alternatives. Let candidate x be the best alternative of the majoritarian group out of chooser´s $\#A{-}k + 1$-top alternatives. Let $B \in A_k$ be a set where x is the chooser best candidate in this set. Since $m \geq q_k$ and by definition of q_k, there is a strategy profile that can be adopted by the majoritarian group that leads the election of x and the minoritarian group is unable to change it. Notice that the majoritarian group will not incentive in changing outcome. Therefore, there exists an strategy profile that sustains x as strong Nash equilibrium outcome.

Suppose that $q_k > m$.

1) Suppose that the tie breaking criterion coincides with the chooser´s preferences over the set of candidates or with the minoritarian group ´s preferences over the set of candidates.

Notice chooser 1-top candidate cannot be considered better than x by one of the group of proposers of voters. Since both groups could impose (since $q_k > m$ and chooser 1-top candidate is also ranked higher than x according to the tie breaking criterion) the inclusion of the chooser 1-top in the selected list, preventing x being elected. In addition, each group has the reverse preference profile of the other group. Thus, x needs to be the chooser´s 1-top candidate. Suppose that every proposer cast a vote for x. Thus, x will be in the selected list and it will be elected. No group can take out x from the selected list by a unilateral deviation since both has size smaller than q_k. Since both group has the reverse preference profile of the other, they do not have incentive in jointly deviating from this strategy profile. Therefore, this strategy profile sustains x as an strong Nash equilibrium outcome.

2) Suppose that the tie breaking criterion coincides with the majoritarian group´s preferences over the set of candidates. Suppose also that $q_k > m \geq q_1 > n - m$.

Notice that $q_k > m$ implies that minoritarian group can impose that the chosen list has

at least one name among k-top candidates according the tie breaking criterion. Notice that $q_1 > n - m$ implies that the majoritarian group can impose that the chosen list be formed by the k-top candidates according the tie breaking criterion. Thus, x need to be the chooser´s best alternative out of the majoritarian group´s k-top candidates. Now, we need to prove that there exists a strategy profile that sustains x as an equilibrium outcome. Suppose the following strategy profile: the majoritarian group adopts a strategy profile that can impose the selection of k-top candidates according the tie breaking criterion and the minoritarian group adopts a strategy profile that can impose the chooser best candidate among the k-top candidates according to the tie breaking criterion. In order to change the outcome, one of the group could try to block the inclusion of x, but neither of them alone manage to do it. Notice also that only the majoritarian group would be able to include another candidate better than x to the chooser. But this candidate would be worse than x for the majoritarian group. Therefore, there exists not coaltion of proposers that has incentive in deviating. Thus, we have proved that there exists a strategy profile that sustains x as an equilibrium outcome.
In order to finish the proof, suppose that $q_k > m > n - m \geq q_1$. Notice that since both groups of voters have size higher than q_1, both groups can impose a list with the chooser´s 1-top candidate. Since each group has the reverse preference profile of the other group. The equilibrium outcome needs to be the chooser´s 1-top candidate. Suppose that every proposer cast a vote for x. Thus, x will be in the selected list and it will be elected. No group can take out x from the selected list by a unilateral deviation since both has size smaller than q_k. Since both group has the reverse preference profile of the other, they do not have incentive in jointly deviating from this strategy profile. Therefore, this strategy profile sustains x as an strong Nash equilibrium outcome. ■

Proof of Proposition 10. Given that $u(\cdot)$ is decreasing on r, we only need to prove that:

1) $F_c(x|k,a)$ first order stochastically dominates $F_p(x|k,a)$ for every $k < \frac{a+1}{2}$;

2) $F_p(x|k,a)$ first order stochastically dominates $F_c(x|k,a)$ for every $k > a - \sqrt[2]{a} + 1$.

By definition of first order stochastic domination, it is sufficient to show that:

1) If $k < \frac{a+1}{2}$ then $F_p(x|k,a) \geq F_c(x|k,a)$ for every $x \in \{1,...,a\}$.

2) If $k > a - \sqrt[2]{a} + 1$ then $F_c(x|k,a) \geq F_p(x|k,a)$ for every $x \in \{1,...,a\}$

Let us first prove that if $k < \frac{a+1}{2}$ then $F_p(x|k,a) \geq F_c(x|k,a)$ for every $x \in \{1,...,a\}$. Take any $k^* \in \{1,...,\frac{a+1}{2} - 1\}$. Since $k^* < \frac{a+1}{2}$, we have that $a - k^* + 1 > k^*$.Thus, $F_p(x|k^*,a) = F_c(x|k^*,a) = 1$ for every $x \in \{a - k^* + 1,...,a\}$ and $F_p(x|k^*,a) = 1 > F_c(x|k^*,a)$ for every $x \in \{k^*,...,a-k^*\}$. Now let us examine the case where $x \in \{1,...,k^* - 1\}$. Consider $x = k^* - 1$.

$F_c(x = k^* - 1|k^*,a) = \frac{k^*-1}{a-k^*+1} < \frac{k^*-1}{k^*}$

$F_p(x = k^* - 1|k^*,a) = \sum_{j=1}^{k^*-1} \frac{\binom{a-j}{a-k^*}}{\binom{a}{a-k^*+1}} = 1 - \frac{1}{\binom{a}{a-k^*+1}} \geq 1 - \frac{1}{\binom{a}{a-1}} \geq 1 - \frac{1}{k^*} = \frac{k^*-1}{k^*}$

Thus, $F_p(x = k^* - 1|k^*,a) > F_c(x = k^* - 1|k^*,a)$.

Consider $x = 1:$

$F_p(x = 1|k^*,a) = \frac{a-k^*+1}{a}$

$F_c(x = 1|k^*,a) = \frac{1}{a-k^*+1}$

Notice that $\frac{a-k+1}{a} > \frac{1}{a-k+1}$ for every $k < a - \sqrt[2]{a} + 1$. Thus, $F_p(x = 1|k^*,a) > F_c(x = 1|k^*,a)$ since $k^* \leq \frac{a+1}{2} < a - \sqrt[2]{a} + 1$.

Given that, by definition, $F_p(x|k^*,a)$ and $F_c(x|k^*,a)$ are strict increasing function in the interval $\{1,...,k^* - 1\}$, $F_p(x = 1|k^*,a) > F_c(x = 1|k^*,a)$ and $F_p(x = k^* - 1|k^*,a) > F_c(x = k^* - 1|k^*,a)$, we have that:

$F_p(x|k,a) > F_c(x|k,a)$ for every $x \in \{1,...,k^* - 1\}$.

Therefore, we have that $F_p(x|k,a) \geq F_c(x|k,a)$ for every $x \in \{1,...,a\}$.

To finish we need to prove that if $k > a - \sqrt[2]{a} + 1$ then $F_c(x|k,a) \geq F_p(x|k,a)$ for every $x \in \{1,...,a\}$.

Take any $k^* \in \{a - \sqrt[2]{a} + 2,...,a\}$.Since $k > a - \sqrt[2]{a} + 1$, we have that $a - k^* + 1 < k^*$.Thus, $F_p(x|k^*,a) = F_c(x|k^*,a) = 1$ for every $x \in \{k^*,...,a\}$ and $F_c(x|k^*,a) = 1 > F_p(x|k^*,a)$ for every $x \in \{a - k^* + 1,...,k^* - 1\}$. Now let us examine the case where $x \in \{1,...,a - k^*\}$.

Consider $x = 1:$

$F_p(x = 1|k^*,a) = \frac{a-k^*+1}{a}$

$F_c(x = 1|k^*,a) = \frac{1}{a-k^*+1}$

Notice that $\frac{a-k+1}{a} < \frac{1}{a-k+1}$ for every $k > a - \sqrt[2]{a} + 1$. Thus, $k^* > a - \sqrt[2]{a} + 1$ implies that $F_c(x = 1|k^*,a) > F_p(x = 1|k^*,a)$.

Given that $F_p(x|k^*,a)$ and $F_c(x|k^*,a)$ are strict increasing function in the $x \in \{1,...,a -$

$k^* + 1\}$, $F_c(x = 1|k^*, a) > F_p(x = 1|k^*, a)$ and $F_c(x = a - k^* + 1|k^*, a) = 1 > F_p(x = a - k^* + 1|k^*, a)$, we have that:
$F_c(x|k^*, a) \geq F_p(x|k^*, a)$ for every $x \in \{1, ..., a - k^* + 1\}$.
Thus, we have that $F_p(x|k, a) \geq F_c(x|k, a)$ for every $x \in \{1, ..., a\}$. Therefore, the proof is established. ■

Proof of Proposition 11. Suppose that $a + 2 - \sqrt{2a+2}$ is an integer and take any strict decreasing and concave utility function $u(\cdot)$. Let $k > a + 2 - \sqrt{2a+2}$ then $k > a - k + 1$ and, by Proposition 9, $E(R_c|k, a) < E(R_p|k, a)$. Notice that $k > a - k + 1$, so it implies that $\{x \in \{1, ..., a\}|Prob(r_c = x|k, a) > 0\} \subset \{x \in \{1, ..., a\}|Prob(r_p = x|k, a) > 0\}$. Thus, $F_c(x|k, a)$ is more concentrated than $F_p(x|k, a)$ and, in additional, it has a smaller mean. Therefore, $E(u(R_c)) > E(u(R_p))$.
Take now any strict decreasing and convex utility function $u(\cdot)$. Take any $k \in [\frac{a+1}{2}, a + 2 - \sqrt{2a+2})$then $k \geq a - k + 1$ and, by Proposition 10, $E(R_p|k, a) < E(R_c|k, a)$. Notice that $k > a - k + 1$, so it implies that $\{x \in \{1, ..., a\}|Prob(r_c = x|k, a) > 0\} \subseteq \{x \in \{1, ..., a\}|Prob(r_p = x|k, a) > 0\}$. Thus, $F_p(x|k, a)$ is at least as concentrated than $F_c(x|k, a)$ and, in additional, it has a smaller mean. Therefore, $E(u(R_p)) > E(u(R_c))$. Notice that for $k < \frac{a+1}{2}$, Proposition 9 states that $E(u(R_p)) > E(u(R_c))$. Therefore, the proof is established. ■

Proof of Proposition 12. First $a > t(a, \gamma_c, \gamma_p) > 1$ comes from the fact that $E(u_p(R_p)|a, k = 1) > E(u_c(R_c)|a, k = 1)$ and $E(u_c(R_c)|a, k = a) > E(u_p(R_p)|a, k = a)$. Thus, we only need to show that for any $k \geq a+2-\sqrt{2a+2}$, we have that $E(u_c(r)|a, k) > E(u_p(r)|a, k)$. First let $k = a + 2 - \sqrt{2a+2}$, then we have that $E(R_c|a, k) = E(R_p|a, k)$ and $Var(R_p|a, k) = \frac{3(\sqrt[2]{2a+2}-1)^2}{(2a+1)} Var(R_c|a, k)$.
$E(u(R_c)|a, k) - E(u(R_p)|a, k) = -\gamma_c(Var(R_c|a, k)) + E(R_c|a, k)^2) + \gamma_p(Var(R_p|a, k))$
$+ E(R_p|a, k)^2) + (\gamma_c - \gamma_p) =$
$= -\gamma_c(Var(R_c|a, k) + E(R_c|a, k)^2) + \gamma_p(\frac{3(\sqrt[2]{2a+2}-1)^2}{(2a+1)} Var(R_c|a, k) + E(R_c|a, k)^2) + (\gamma_c - \gamma_p)$
$= (\gamma_p - \gamma_c)(E(R_c|a, k)^2 - 1) + (\gamma_p \frac{3(\sqrt[2]{2a+2}-1)^2}{(2a+1)} - \gamma_c)Var(R_c|a, k)$.
Thus, $E(u(R_c)|a, k) - E(u(R_p)|a, k) > 0$ since $\frac{3(\sqrt[2]{2a+2}-1)^2}{(2a+1)} > 1$, $E(R_c|a, k)^2 > 1$ and $0 < \gamma_c \leq \gamma_p$. So, it is true for any $k \geq a+2-\sqrt{2a+2}$, thus $1 < t(a, \gamma_c, \gamma_p) < a+2-\sqrt{2a+2}$.
Now, if $\gamma_c \geq \frac{3(\sqrt[2]{2a+2}-1)^2}{(2a+1)} \gamma_p > 0$, we have that $E(u_p(r)|a, k) > E(u_c(r)|a, k)$ under $k = a + 2 - \sqrt{2a+2}$. So, it is true for any $k \leq a + 2 - \sqrt{2a+2}$, thus $a > t(a, \gamma_c, \gamma_p) > a+2-\sqrt{2a+2}$. The proof of the other cases is similar and for this reason is ommited. ■

Proof of Proposition 15. Equation 7 below gives the formula of the sum of agents´expected utilities:

$$wE(u_p(R_p)|k,a)+(1-w)E(u_c(R_c)|k,a)=-\frac{w(a+1)}{(a-k+2)}-\frac{w\gamma_p(a+k+1)(a+1)}{(a-k+2)(a-k+3)}+w\gamma_p$$

$$-(1-w)\frac{(a-k+2)}{2}-\frac{(1-w)(a-k+2)}{2}-\frac{\gamma_c(1-w)(a-k+2)(2a-2k+3)}{6}+(1-w)\gamma_c \qquad (13)$$

Notice also that expression (13) implies expressions (14) and (15) below:

$$wE(u_p(R_p)|k,a)-wE(u_p(R_p)|k-1,a)=-\frac{w(a+1)}{(a-k+3)(a-k+2)}-\frac{w\gamma_p(3a+k+4)(a+1)}{(a-k+2)(a-k+3)(a-k+4)} \qquad (14)$$

$$(1-w)E(u_c(R_c)|k,a)-(1-w)E(u_c(R_c)|k-1,a)=\frac{(1-w)(4a\gamma_c-4k\gamma_c+9\gamma_c+3)}{6} \qquad (15)$$

After summing up (14) and (15), we have that:

$$[wE(u_p(R_p)|k,a)+(1-w)E(u_c(R_c)|k,a)]-$$
$$[wE(u_p(R_p)|k-1,a)+(1-w)E(u_c(R_c)|k-1,a)]=$$

$$=-\frac{w(a+1)}{(a-k+3)(a-k+2)}-\frac{w\gamma_p(3a+k+4)(a+1)}{(a-k+2)(a-k+3)(a-k+4)}+\frac{(1-w)(4a\gamma_c-4k\gamma_c+9\gamma_c+3)}{6} \qquad (16)$$

Hence with the help of expression (16) it can be easily proved that the optimal k can be characterized as follows: $\widehat{k}$ is the largest $k\acute{} \in \{1,...,a\}$ such that $\frac{(a+1)(a-k'+4)+\gamma_p(3a+k'+4)(a+1)}{(a-k+3)(a-k'+2)(4a\gamma_c-4k'\gamma_c+9\gamma_c+3)} \geq \frac{(1-w)}{w}\frac{1}{6}$ and $\frac{(a+1)(a-k'+4)+\gamma_p(3a+k'+4)(a+1)}{(a-k+3)(a-k'+2)(4a\gamma_c-4k'\gamma_c+9\gamma_c+3)} \leq \frac{(1-w)}{w}\frac{1}{6}$ for any $k > k\acute{}$.
Moreover if $\frac{(a+1)(a-k'+4)+\gamma_p(3a+k'+4)(a+1)}{(a-k+3)(a-k'+2)(4a\gamma_c-4k'\gamma_c+9\gamma_c+3)} = \frac{(1-w)}{w}\frac{1}{6}$ then $k\acute{}$ is a twin-dips. ■

Proof of Proposition 16. First notice that for every k we have that:
$E(u_p(R_p)|a,k)E(u_c(R_c)|a,k)=\frac{a+1}{2}$
The equality above implies that
$(E(u_p(R_p)|a,k)+E(u_c(R_c)|a,k))^2=E(u_p(R_p)|a,k)^2+E(u_c(R_c)|a,k)^2+(a+1)$
The expression above implies that, given that $E(u_p(R_p)|a,k)+E(u_c(R_c)|a,k)<0$, a $k\in\{1,...a\}$ maximizes $E(u_p(R_p)|a,k)+E(u_c(R_c)|a,k)$ if and only if it minimizes $E(u_p(R_p)|a,k)^2+E(u_c(R_c)|a,k)^2$.
Notice also that a
$(E(u_p(R_p)|a,k)-E(u_c(R_c)|a,k))^2=E(u_p(R_p)|a,k)^2+E(u_c(R_c)|a,k)^2-(a+1)$.
The expression above implies that a $k\in\{1,...a\}$ maximizes $E(u_p(R_p)|a,k)^2+E(u_c(R_c)|a,k)^2$ if and only if it maximizes $(E(u_p(R_p)|a,k)-E(u_c(R_c)|a,k))^2$.

Therefore, a $k \in \{1, ...a\}$ maximizes $E(u_p(R_p)|a,k) + E(u_c(R_c)|a,k)$ if and only if minimizes $|E(u_p(R_p)|a,k) - E(u_c(R_c)|a,k))|$. ■

Proof of Proposition 18. By Proposition 8, if $m \geq q_k > n-m$ then the strong Nash equilibrium outcome is the best alternative of individuals in the majoritarian group out of chooser´ s $(\#\mathbf{A} - k + 1)$-top alternatives;

Thus, the expected ranking of the equilibrium outcome according to the majoritarian group´s distribution fo preferences:

$E(u_1(r_1)|m \geq q_k > n-m) = \frac{(a+1)}{a-k+2}$

The expected utility of the minoritarian group´s distribution fo preferences is: $E(u_2(r_2)|m \geq q_k > n-m) = -(a+1) - E(u_1(r_1)|m \geq q_k > n-m)$. Thus,

$E(u_2(r_2)|m \geq q_k > n-m) = -\frac{(a+1)(a-k+1)}{a-k+2}$

The expected ranking of the equilibrium outcome according to the chooser´s distribution of preferences is:$\frac{a-k+2}{2}$

$E(u_c(R_c)|m \geq q_k > n-m) = -\frac{a-k+2}{2}$

Suppose $q_k > m > n-m \geq q_1$. By Proposition 8, if $q_k > m \geq q_1 > n-m$ then the strong Nash equilibrium outcome is the chooser´s best alternative out of the majoritarian group´s k-top candidates. Thus:

$E(u_1(r_1)|q_k > m > n-m \geq q_1) = -\frac{k+1}{2}$

$E(u_2(r_2)|q_k > m > n-m \geq q_1) = -\frac{2a-k+1}{2}$

$E(u_c(R_c)|q_k > m > n-m \geq q_1) = -\frac{a+1}{k+1}$

By Proposition 8, if $q_k > m > n-m \geq q_1$ then the equilibrium outcome is the chooser´s top alternative.Thus, the chooser has all the power:

$E(u_1(r_1)|q_k > m > n-m \geq q_1) = -\frac{(a+1)}{2}$

$E(u_2(r_2)|q_k > m > n-m \geq q_1) = -\frac{(a+1)}{2}$

$E(u_c(R_c)|q_k > m > n-m \geq q_1) = -1$. ■